Make sure to fill out the grading sheet completely including your name, exam flavor and UIN. You are allowed to write and work on this exam copy, but your answers must be bubbled in on the grading sheet to receive credit. Your bubbled responses are the only responses that will be considered for the grade.

Physics 207 Comprehensive Exam – Flavor 1

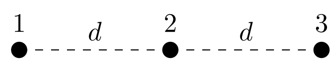
Problem 1 (6 points) Three electrons are held at rest in a line and evenly spaced by a distance d as shown in the figure below. Electrons 1 and 3 are simultaneously released while electron 2 in the middle is held at rest. What are the speeds of electrons 1 and 3 when they are an effectively infinite distance away from their starting point?

(A) $\sqrt{\frac{ke^2}{2m_e d}}$

(B) $\sqrt{\frac{ke^2}{m_e d}}$

(C) $\sqrt{\frac{3ke^2}{2m_e d}}$

(D) $\sqrt{\frac{2ke^2}{m_e d}}$



(E) $\sqrt{\frac{5ke^2}{2m_e d}}$

(F) $\sqrt{\frac{3ke^2}{m_e d}}$

(G) $\sqrt{\frac{4ke^2}{m_e d}}$

(H) $\sqrt{\frac{5ke^2}{m_e d}}$

Problem 2 (7 points) An insulating sphere has a radius R and has a non-uniform charge density $\rho(r) = \alpha r$. What is the magnitude of the electric field at all points outside the sphere where $r > R$?

(A) $\frac{\alpha R^2}{4\epsilon_0}$

(B) $\frac{\alpha R^2}{8\epsilon_0}$

(C) $\frac{\alpha r^4}{4\pi\epsilon_0}$

(D) $\frac{\alpha r^2}{8\pi\epsilon_0}$

(E) $\frac{\alpha R^2}{4\pi\epsilon_0 r^2}$

(F) $\frac{\alpha R^2}{8\pi\epsilon_0 r^2}$

(G) $\frac{\alpha R^4}{4\epsilon_0 r^2}$

(H) $\frac{\alpha R^4}{8\epsilon_0 r^2}$

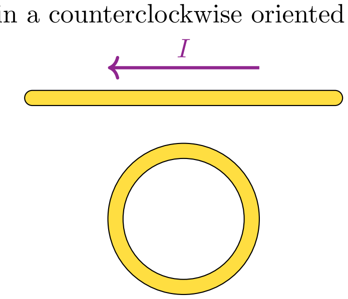
Problem 3 (6 points) An RC-circuit is composed of a 150 kΩ resistor, a 330 μF capacitor and a battery. When charging, it takes 25.0 s for the charge on the capacitor to reach 4.58 mC. What is the emf supplied by the battery? Assume the battery has a negligible internal resistance.

(A) 50.0 V
(B) 42.0 V
(C) 35.0 V
(D) 23.0 V
(E) 12.0 V

Problem 4 (7 points) An object with a charge of $q = +3$ C has a velocity vector $\vec{v} = 200\hat{x} - 300\hat{y}$ in the presence of an electric and a magnetic field. The magnetic field is known to be $\vec{B} = 5\hat{z}$ and the net force vector that the proton feels is $\vec{F} = 1500\hat{x} - 1500\hat{y}$. What is the electric field at the location of the charge?

(A) $\vec{E} = 6000\hat{x} + 1500\hat{y}$
(B) $\vec{E} = 6000\hat{x} - 4500\hat{y}$
(C) $\vec{E} = 4000\hat{x} + 1000\hat{y}$
(D) $\vec{E} = 4000\hat{x} - 3000\hat{y}$
(E) $\vec{E} = 2000\hat{x} + 500\hat{y}$
(F) $\vec{E} = 2000\hat{x} - 1500\hat{y}$

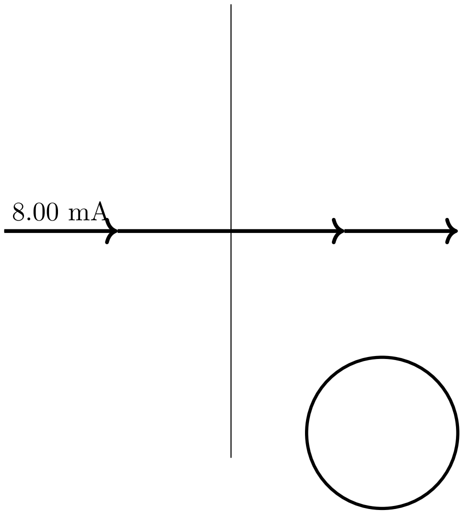
Problem 5 (3 points) There is a constant current moving to the left in the long, straight wire. What direction should the circular loop move if the motion should result in a counterclockwise oriented induced current in the loop?



(A) Left
(B) Right
(C) Up
(D) Down

Problem 6 (7 points) A long straight wire is carrying an 8.00-mA current to the right along the x-axis in the positive x-direction. There is a coil of 25 loops that have a radius of 3.00 m with the center at the location $(x, y) = (6.00, -8.00)$ where these values are in meters. What is the magnitude of current necessary so that the net magnetic field at the center of the loop is exactly zero?

(A) 10.6 μA
(B) 19.1 μA
(C) 38.2 μA
(D) 63.7 μA
(E) 71.9 μA
(F) 102 μA

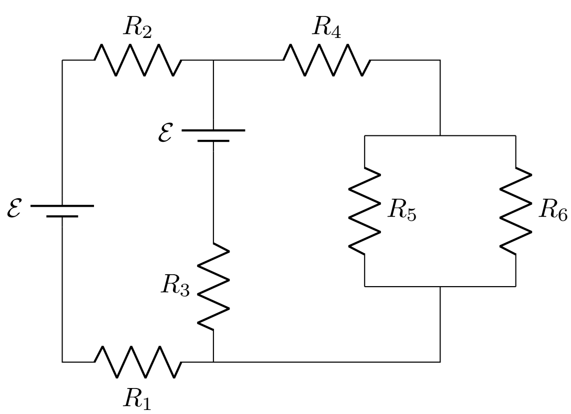


Problem 7 (3 points) What is the direction of the current you found in the previous problem?

(A) Clockwise
(B) Counterclockwise

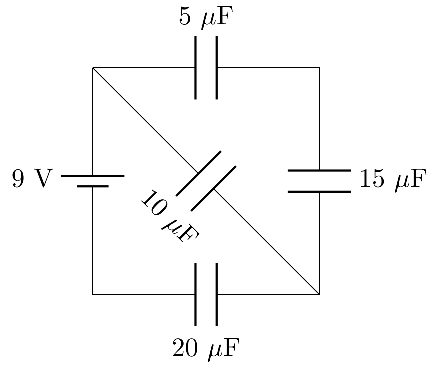
Problem 8 (6 points) In the circuit below all resistors have an identical resistance R (the indices are only for identification). Both batteries have identical emf $\mathcal{E}$. The current through the resistor marked R_1 is I to the left and the current through R_6 is $\frac{3}{2}I$ downwards. What is the emf in terms of I and R?

(A) $\mathcal{E} = \frac{1}{2}IR$
(B) $\mathcal{E} = IR$
(C) $\mathcal{E} = \frac{7}{2}IR$
(D) $\mathcal{E} = \frac{9}{2}IR$
(E) $\mathcal{E} = \frac{13}{2}IR$



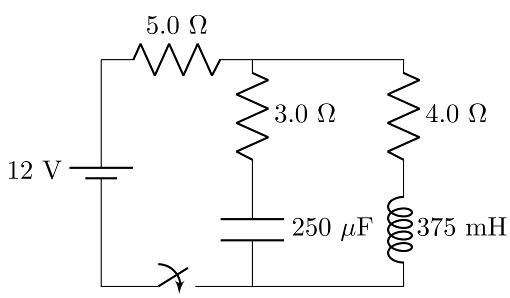
Problem 9 (7 points) What is the total charge stored in the capacitor network below due to the 9 V battery?

(A) 8.15 μC
(B) 26.7 μC
(C) 73.3 μC
(D) 157 μC
(E) 240 μC
(F) 338 μC



Problem 10 (6 points) In the circuit below, what is the current through the 3.0 Ω resistor at the instant the switch is closed?

(A) 0.67 A
(B) 1.33 A
(C) 1.50 A
(D) 1.79 A
(E) 2.67 A



Problem 11 (6 points) The distance between two charges is a. If the charges are $-4q$ and $-q$, how far away from the $-4q$ charge is the net electric field zero along the line between the charges?

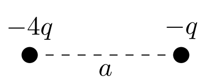
(A) $\frac{1}{2}a$

(B) $\frac{3}{5}a$

(C) $\frac{2}{3}a$

(D) $\frac{4}{5}a$

(E) $\frac{5}{6}a$



Problem 12 (6 points) A positive charge q is uniformly distributed in a quarter-circumference of a circle with radius a in quadrant 1 as shown below. An identical arc is also in quadrant 3. What is the magnitude of the net electric field at the origin?

(A) $\frac{2kq}{a}$

(B) $\frac{kq}{a}$

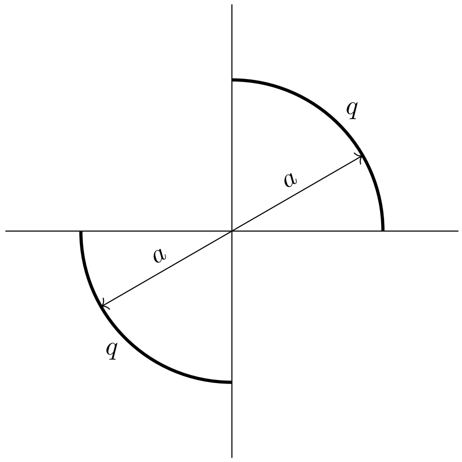
(C) $\frac{kq}{\pi a}$

(D) $\frac{2kq}{a^2}$

(E) $\frac{2\sqrt{2}kq}{\pi a^2}$

(F) $\frac{4\sqrt{2}kq}{\pi a^2}$

(G) 0



Problem 13 (6 points) What is the total electric potential at the origin due to the arcs of charge in the previous problem?

(A) $\frac{2kq}{a}$

(B) $\frac{kq}{a}$

(C) $\frac{kq}{\pi a}$

(D) $\frac{2kq}{a^2}$

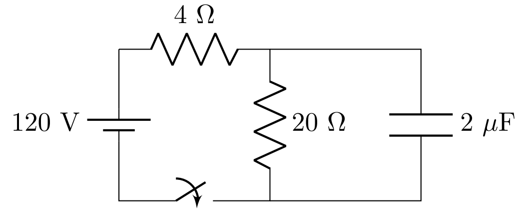
(E) $\frac{2\sqrt{2}kq}{\pi a^2}$

(F) $\frac{4\sqrt{2}kq}{\pi a^2}$

(G) 0

Problem 14 (6 points) What is the voltage across the capacitor if the switch has been closed for a long time?

(A) 120 V
(B) 100 V
(C) 60 V
(D) 20 V
(E) 0 V



Problem 15 (6 points) A square loop of wire with side length ℓ exists in the xy-plane. There is a magnetic field of strength B_0 oriented in the positive z-direction. At $t = 0$ the magnetic field changes to have a magnitude of $3B_0$ and also changes so that it points in the negative z-direction. What is the magnitude of the average induced emf due to this change if it happens over a span of t seconds?

(A) $\mathcal{E} = 5\dfrac{B_0\ell^2}{t}$
(B) $\mathcal{E} = 4\dfrac{B_0\ell^2}{t}$
(C) $\mathcal{E} = 3\dfrac{B_0\ell^2}{t}$
(D) $\mathcal{E} = 2\dfrac{B_0\ell^2}{t}$
(E) $\mathcal{E} = 1\dfrac{B_0\ell^2}{t}$
(F) $\mathcal{E} = 0$

Problem 16 (6 points) The average intensity of light that hits Jupiter from the sun is about 51.4 W/m^2. What is the magnitude of the magnetic field of this light?

(A) 6.56×10^{-7} T
(B) 4.64×10^{-7} T
(C) 2.43×10^{-7} T
(D) 1.77×10^{-7} T

Problem 17 (6 points) At a given instant in time, the magnetic flux through circuit A due to circuit B is 40.0 mWb and the magnetic flux through circuit B due to circuit A is 12.0 mWb. You know that circuit A has a current of 250 mA and contains 500 loops. Circuit B contains 750 loops. What is the mutual inductance of this system?

(A) 320 H
(B) 200 H
(C) 96 H
(D) 80 H
(E) 36 H

Useful Constants:
Acceleration due to gravity: $g = 9.80$ m/s^2
Basic unit of charge: $e = 1.6 \times 10^{-19}$ C
Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
Mass of proton/neutron: $m_p = 1.67 \times 10^{-27}$ kg
Coulomb constant: $k = 8.99 \times 10^9$ Nm^2/C^2
Permittivity of free space: $\epsilon_0 = 1/(4\pi k) = 8.85 \times 10^{-12}$ C^2/Nm^2
Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ Tm/A
Speed of light in a vacuum: $c = 3 \times 10^8$ m/s
Planck's Constant: $h = 6.626 \times 10^{-34}$ Js
eV to joule conversion: 1 eV = 1.6×10^{-19} J
kilowatt-hour to joule conversion: 1 kW·hr=3.6×10^6 J
Atomic Mass Unit: 1 u = 1.66054×10^{-27} kg = 931.5 MeV/c^2

Vector Concepts:
Unit Vector: $\hat{r} = \frac{\vec{r}}{r}$
Gradient: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
Dot Product: $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$
Dot Product: $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$
Cross Product:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$
$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{x} - (A_xB_z - A_zB_x)\hat{y} + (A_xB_y - A_yB_x)\hat{z}$$

Sample Indefinite Integrals:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) + c$$
$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + c$$
$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}} + c$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c(n \neq -1)$$
$$\int \frac{dx}{x} = \ln(x) + c$$
$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + c$$

SI Prefixes:
T=$\times 10^{12}$, G=$\times 10^9$, M=$\times 10^6$, k=$\times 10^3$
c=$\times 10^{-2}$, m=$\times 10^{-3}$
μ=$\times 10^{-6}$, n=$\times 10^{-9}$, p=$\times 10^{-12}$, f=$\times 10^{-15}$

Useful integral relationships:
Spherical: $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$
Cylindrical (with constant r): $dV = \pi r^2 \mathrm{d}z$
Cylindrical (with constant z): $dV = z 2\pi r \mathrm{d}r$
Cylindrical (with constant r): $dA = 2\pi r \mathrm{d}z$
Cylindrical (with constant z): $dA = 2\pi r \mathrm{d}r$

Geometry:
Surface Area of a Sphere: $A = 4\pi r^2$
Volume of a Sphere: $V = \frac{4}{3}\pi r^3$
Area of curved region of a cylinder: $A = 2\pi rh$
Volume of a cylinder: $V = \pi r^2 h$

Physics 1 Concepts:
Work: $W = \int \vec{F} \cdot d\vec{\ell}$
Potential Energy of conservative force:
$W_{cons} = -\Delta U$
Kinetic Energy: $K = \frac{1}{2}mv^2$
Momentum: $\vec{p} = m\vec{v}$

Chapter 21:

Coulomb's Law [N]: $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$

Force due to an electric field [N]: $\vec{F} = q\vec{E}$

E Field due to a pt. charge [N/C]: $\vec{E} = \frac{kq}{r^2}\hat{r}$

E Field due to a continuous charge dist. [N/C]:
$\vec{E} = \int \frac{kdq}{r^2}\hat{r}$

Electric dipole moment [Cm]: $\vec{p} = q\vec{d}$

Torque on an electric dipole [Nm]: $\vec{\tau} = \vec{p} \times \vec{E}$

Electric pot. energy stored in electric dipole [J]:
$U = -\vec{p} \cdot \vec{E}$

Chapter 22:

Electric Flux [Vm or Nm^2/C]: $\Phi_E = \int \vec{E} \cdot d\vec{A}$

Electric Flux when E and θ are const. on the surface: $\Phi_E = EA\cos\theta$

Gauss's Law (vacuum): $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$

Chapter 23: The below equations generally but not always assume that $V(\infty) = 0$ and/or $U(\infty) = 0$.

Elec. pot. energy between 2 pt charges [J]:
$U = \frac{kq_1q_2}{r}$

Elec. pot. difference btw. two locations [V or J/C]:
$\Delta V = \frac{\Delta U}{q}$ (or often) $V = \frac{U}{q}$

Electric potential due to a point charge [V]:
$V = \frac{kq}{r}$

Electric potential due to a charge dist. [V]:
$V = \int \frac{kdq}{r}$

Relating $\vec{E}$ and V: $\vec{E} = -\vec{\nabla}V$

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$$

Chapter 24:

Capacitance [F]: $Q = CV$

Capacitance for Parallel Plates [F]: $C = \frac{\kappa\epsilon_0 A}{d}$

Energy stored in a capacitor [J]: $U = \frac{1}{2}CV^2$

E field energy density [J/m^3]: $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$

Definition of Dielectric Constant: $C = \kappa C_0$

Eff. Cap. (series) [F]: $\frac{1}{C_{eff}} = \sum_i \frac{1}{C_i}$

Eff. Cap. (parallel) [F]: $C_{eff} = \sum_i C_i$

Chapters 25 and 26:

Electric Current [A]: $I = \frac{dq}{dt}$

I from current density [A]: $I = \int \vec{j} \cdot d\vec{A}$

j of uniform current [A/m^2]: $|\vec{j}| = \frac{I}{A}$

j for charges in motion [A/m^2]: $\vec{j} = nq\vec{v}_d$

Ohm's Law: $\vec{E} = \rho\vec{j}$

Ohm's Law: $\Delta V = IR$ (or often just) $V = IR$

Resistivity and conductivity: $\rho = \frac{1}{\sigma}$

Resistance of a wire [Ω]: $R = \frac{\rho\ell}{A}$

Resistance of an object [Ω]: $R = \int \frac{\rho(x)dx}{A(x)}$

Power in a circuit element [W]:
$P = I\Delta V$ (or often) $P = IV$

Eff. Res. (series) [Ω]: $R_{eff} = \sum_i R_i$

Eff. Res. (parallel) [Ω]: $\frac{1}{R_{eff}} = \sum_i \frac{1}{R_i}$

Time constant for an RC-circuit [s]: $\tau = RC$

Charge on a charging capacitor [C]:
$q(t) = q_{\max}(1 - e^{-t/\tau})$

Charge on a discharging capacitor [C]:
$q(t) = q_0 e^{-t/\tau}$

Current in an RC-circuit [A]: $I(t) = I_0 e^{-t/\tau}$

Chapter 27:

Mag. Force on a moving q [N]: $\vec{F} = q\vec{v} \times \vec{B}$
Mag. Force on a current-carrying conductor [N]:
$\vec{F} = I \int d\vec{\ell} \times \vec{B}$

R of q's path in a B field [m]: $R = \frac{mv}{|q|B}$

Magnetic Dipole Moment [Am^2]: $\vec{\mu} = I\vec{A}$

Torque on current loops [Nm]: $\vec{\tau} = N\vec{\mu} \times \vec{B}$

Mag. pot. energy in a magnetic dipole [J]:
$U = -N\vec{\mu} \cdot \vec{B}$

Chapter 28:

Biot-Savart Law (2 forms):

B made by a moving charge [T]: $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$

B made by any current [T]: $\vec{B} = \int \frac{\mu_0}{4\pi}\frac{Id\vec{\ell} \times \hat{r}}{r^2}$

B made by a long straight wire in a vacuum [T]:
$B = \frac{\mu_0 I}{2\pi r}$

B made by N loops, w/ radius R, on the axis,
z from the center (vacuum) [T]: $B = \frac{N\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$

B made inside a solenoid: $B = \mu_0 K_m \frac{N}{\ell} I$

Chapter 29:

Magnetic Flux [Wb]: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Magnetic Flux when B and θ are const.
on the surface: $\Phi_B = BA\cos\theta$

Faraday's Law [V]: $\mathcal{E} = -N\frac{d\Phi_B}{dt}$

Motional emf [V]: $\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$

Induced E Fields: $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

Displacement current [A]: $i_d = \epsilon_0 \frac{d\Phi_E}{dt}$

General Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I_{\text{encl}} + i_d)$

Chapter 30:

emf and Mutual Inductance:
$\mathcal{E}_1 = -M\frac{di_2}{dt}$, $\mathcal{E}_2 = -M\frac{di_1}{dt}$

Mut. Inductance [H]: $M = \frac{N_2\Phi_{B,2}}{i_1} = \frac{N_1\Phi_{B,1}}{i_2}$

emf and Self Inductance: $\mathcal{E} = -L\frac{di}{dt}$

Self Inductance [H]: $L = \frac{N\Phi_B}{i}$

Inductance of a Solenoid [H]: $L = \frac{\mu_0 N^2 A}{\ell}$

Magnetic energy stored in an inductor [J]:
$U = \frac{1}{2}LI^2$

B field energy density [J/m^3]: $u_B = \frac{1}{2\mu_0}B^2$

Time Constant in an RL-Circuit [s]: $\tau = L/R$

Current growth in an RL-Circuit [A]:
$i(t) = I_{max}(1 - e^{-t/\tau})$

Current decay in an RL-Circuit [A]:
$i(t) = I_0 e^{-t/\tau}$

Angular frequency of the oscillation
in an LC-Circuit [rad/s]: $\omega = \frac{1}{\sqrt{LC}}$

q on a capacitor in an ideal LC-Circuit [C]:
$q(t) = q_{max}\cos(\omega t + \phi)$

Chapter 32:

Wave speed [m/s]: $v = f\lambda$

Wave number [1/m]: $k = \frac{2\pi}{\lambda}$

E-B relationship: $E = cB$

Speed of light (vacuum): $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

Speed of light (medium): $v = \frac{c}{\sqrt{\kappa K_m}}$

Poynting Vector [W/m^2]: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$

Intensity of a wave in vacuum [W/m^2]:
$I = S_{\text{av}} = \frac{1}{2}uc = \frac{P}{A}$

Total EM energy density [J/m^3]: $u = u_E + u_B$