SOLUTIONS

Physics 208 - Exam I

Spring 2018 (all sections) - February 12, 2018.

Please fill out the information and read the instructions below, but
do not open the exam until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 6 numbered (3 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. **Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!**
5. You will be allowed to use only non-programmable calculators on this exam.
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

Fill out the information below and sign to indicate your understanding of the above rules

Name:______________________________ UIN:____________________

(please print legibly)

Signature:____________________________ Section Number:______________

Instructor: Webb Saslow
(circle one)

A. A charged line segment of length L and non-uniform linear charge density $\lambda(x) = A\,x$, where A is a constant, and $x > 0$ is measured from the left end of the charge segment in meters. The segment is completely contained inside a sphere of radius R.

What are the units of the constant A?

λdl has units of Coulombs ; ~~$\lambda(x)$ has units~~ C/m = unit A * unit x

$\boxed{\text{unit for } A = C/m^2}$

How much charge is enclosed in the sphere?

$$Q_{TOTAL} = \int_0^L \lambda(x)dx = \int Ax\,dx = \frac{Ax^2}{2}\Big|_0^L = \boxed{\frac{AL^2}{2} = Q_{TOTAL}}$$

What is the total flux of the electric field through the surface of this sphere?

From Gauss's Law $\qquad \Phi_{TOTAL} = \oint \vec{E}\cdot d\vec{A} = \frac{q_{en}}{\epsilon_0}$

So the total flux ~~through the surface~~

$$\boxed{\Phi_{TOTAL} = \frac{q_{en}}{\epsilon_0} = \frac{AL^2}{2\epsilon_0}}$$

LO	P	F
5.1		
5.2		
7.1		
5.3		
15.1		
16.1		
3.1		

B. An electron is held at rest at the origin in the potential $V(y) = -5y$, for y in meters and V in volts. The electron is now released from rest.

Find the velocity of the electron when it reaches the point (0, -2.0 cm).

Work done on electron = ΔKE electron ; $\quad -q\Delta V = -q(-5(-.02\,m)) = -q(.1V)$

So $\quad +(1.6\times10^{-19})C\,(.1V) = \frac{1}{2}(9.11\times10^{-31}\,kg)\,v^2$

Solving for $\quad v^2 = \sqrt{\frac{2(1.6\times10^{-19}C)(.1V)}{9.11\times10^{-31}\,kg}}$ $\quad \boxed{1.87\times10^5\,m/s = v}$

LO	P	F
21.1		
21.2		
21.3		
25.1		
7.2		
11.1		
3.2		

From this potential function, what is the acceleration that the electron experiences at the point (0, -2.0 cm)?

$$E_y = -\frac{\partial V(y)}{\partial y} = +5\,N/C$$

So $\quad \boxed{a_y = \frac{F_y}{m} = \frac{qE_y}{m} = \frac{(1.6\times10^{-19}C)(5N/C)}{9.11\times10^{-31}\,kg} = 8.78\times10^{11}\,m/s^2}$

C. A quarter and a nickel are placed on separate insulators. The quarter has a charge of 15.0 X 10^{-9} C and the nickel has an unknown charge. They are now connected by a thin insulated but conducting piece of wire which is then removed.
The quarter now has a charge of 4.5 X 10^{-9} C and the nickel has a charge of 3.75 X 10^{-9} C. What was the initial charge of the nickel?

total charge conserved $= Q_Q^i + Q_N^i = Q_Q^f + Q_N^f$

$q_{TOTAL} = Q_Q^f + Q_N^f = 4.5\times10^{-9}\,C + 3.75\times10^{-9}\,C$

$= 8.25\times10^{-9}\,C$

so $8.25\times10^{-9}\,C = 15.0\times10^{-9} + Q_N^i$; $\boxed{\text{solving for } Q_N^i = -6.75\times10^{-9}\,C}$

The voltage of the quarter is found to be 7.8 V relative to a reference point A, on a door knob, while the voltage of the nickel is measured to be 4.8 V relative to a second reference point B, on the top of a table. Find $V_A - V_B$.

$V_Q - V_A = 7.8V$ $\qquad$ $V_N - V_B = 4.8V$

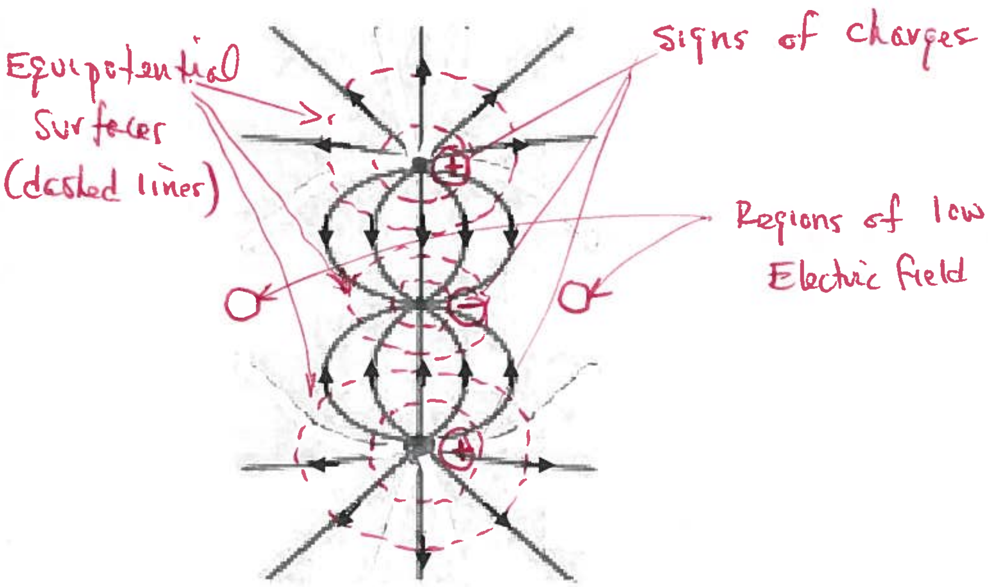
so $V_A = V_Q - 7.8V$ $\qquad$ so $V_B = V_N - 4.8V$

LO	P	F
5.4		
19.1		
22.1		
22.2		
3.3		

to find $V_A - V_B = (V_Q - 7.8) - (V_N - 4.8)$

$$\boxed{= \underbrace{(V_Q - V_N)}_{= 0!} + (4.8 - 7.8) = -3.0V = V_A - V_B}$$

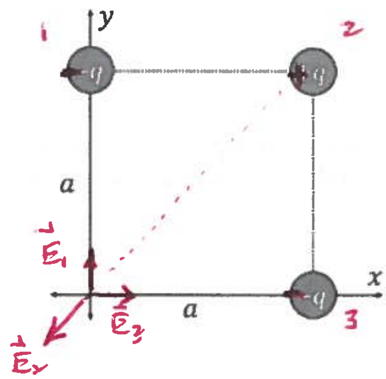
D. The figure below shows some of the electric field lines due to this collection of three charges lined up along the y-axis. All three charges have the same magnitude. A) What are the signs of the three charges and be sure to give your reason. B) Indicate in the figure at what point(s) is the electric field the smallest. Explain your reasoning. C) In the figure, sketch at least two equipotential surfaces around each charge.



LO	P	F
13.1		
13.2		
13.3		
13.4		
27.1		
27.2		
27.3		

Problem I.

Three point charges are fixed at three corners of a square with sides of length ***a***, as shown in the figure. (Express all answers in terms of known values ***q***, ***Q*** and ***a***.)



A. What is the electric field (direction and magnitude) produced by this system of three charges at the origin?

$$\vec{E}_1 = \frac{kq}{a^2}\hat{j} \; ; \; \vec{E}_2 = \frac{kq}{2a^2}(-\cos 45\,\hat{i} - \sin 45\,\hat{j}) \; ; \; \vec{E}_3 = \frac{kq}{a^2}\hat{i}$$

$$\vec{E}_{Total} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \boxed{\frac{kq}{a^2}\left[\left(1 - \frac{.707}{2}\right)\hat{i} + \left(1 - \frac{.707}{2}\right)\hat{j}\right]}$$

B. What is the force (direction and magnitude) exerted by the three ~~positive~~ point charges ***q*** located at the corners of the square on the fourth positive charge ***Q*** positioned at the origin?

$$\vec{F}_Q = Q\vec{E}_{TOTAL} = \boxed{Q\left\{\frac{kq}{a^2}\left[\left(1 - \frac{.707}{2}\right)\hat{i} + \left(1 - \frac{.707}{2}\right)\hat{j}\right]\right\}}$$

LO	P	F
10.1		
10.2		
10.3		
11.2		
9.1		
9.2		
20.1		
20.2		
20.3		
3.4		

C. What is the electric potential energy of the initial 3 charge system?

U_{TOTAL} for 3 charges

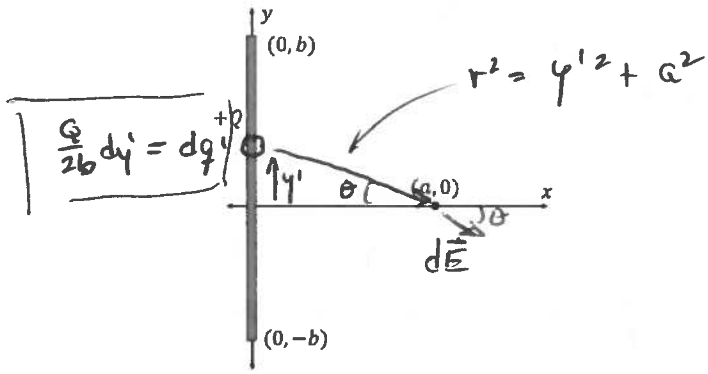
$$= \frac{kq_1q_2}{r_{12}} + \frac{k\,q_1q_3}{r_{13}} + \frac{kq_1q_2}{r_{23}}$$

$$= \frac{kq^2}{a}\left\{-1 + \frac{1}{\sqrt{2}} - 1\right\}$$

$$\boxed{= \frac{kq^2}{a}\left\{\frac{1}{\sqrt{2}} - 2\right\}}$$

Problem II.

A charge ***Q*** (positive) is uniformly distributed along the y-axis from y=-***b*** to y=***b***. A second point charge ***q*** is located at the point (a,0). (Express all answers in terms of known values ***q, Q, b,*** and ***a***.)



A. What is the electric field (magnitude and direction) produced by the uniformly distributed charge ***Q*** at the position of the point charge ***q***, (a, 0)?

From Symmetry there will only be a net x-component for the field at a.

Solving $E_x = \int_{-b}^{+b} \frac{K\,dq'}{(y'^2+a^2)} \left(\frac{a}{(y'^2+a^2)^{1/2}}\right) = \frac{KQ\,a}{2b}\int_{-b}^{+b} \frac{dy'}{(y'^2+a^2)^{3/2}} = \frac{KQa}{2b}\left[\frac{y'}{a^2(y'^2+a^2)^{1/2}}\right]_{-b}^{+b}$

$$\boxed{E_x = \frac{KQ}{a(b^2+a^2)^{1/2}}}$$

B. What is the force (magnitude and direction) exerted by the uniformly distributed charge ***Q*** on the point charge ***q***?

$$\vec{F}_q = q\,\vec{E}_Q = q\,\frac{KQ}{a(b^2+a^2)^{1/2}}\,\hat{i}$$

C. What would be the force exerted on the point charge ***q*** if the uniformly distributed charge is **replaced** by a point charge ***Q*** located at the origin? Compare the result to the force found in B. State clearly in which case (B or C) the force is greater and show your reasoning mathematically.

LO	P	F
12.1		
12.2		
12.3		
7.3		
7.4		
11.3		
11.4		
11.5		
11.6		
3.5		

For 2 point charges Q + q $\quad \vec{F}_{Qq} = \frac{KQq}{a^2}\,\hat{i}$

for the line charge and pt charge found in b) is smaller than the force for 2 point charges.

Problem III.

A very long cylinder with a radius of **a** and a charge density $\rho = \rho_o\, (r^2/a^2)$ is placed inside a conducting cylindrical shell of radius **b** and thickness **t**. For a location that is very far from the depicted end of this cylinder, find the following in terms of the quantities given, **a, b, t,** and ρ_o:

a) Find the electric field for the region $r < a$.

b) Find the electric field for the region $a < r < b$.

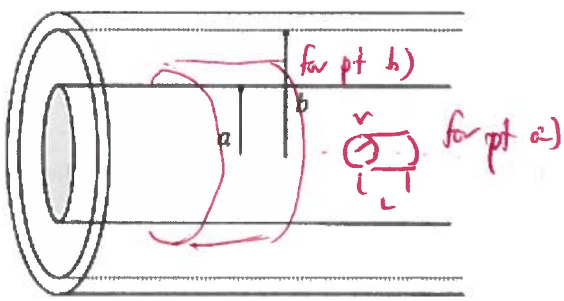
c) Find the electric field for the region $b < r < b + t$.

d) Find the electric field for the region $b + t < r$.

e) Plot E(r).

Using Gauss's Law

$$\oint \vec{E} \cdot dA = \frac{q_{en}}{\epsilon_0}$$



a) $q_{en} = 2\pi \int_0^r L\rho(r)\, r\,dr = 2\pi L\rho_0 \int \left(\frac{r^2}{a^2}\right) r\,dr = \frac{2\pi L\rho_0}{a^2}\int r^3\,dr = \frac{2\pi L\rho_0}{a^2}\left(\frac{r^4}{4}\right)$

then $E(r) = q_{en}/2\pi r\epsilon_0 L = \left(\frac{2\pi L\rho_0}{a^2}\right)\left(\frac{1}{2\pi L\epsilon_0}\right)\frac{1}{r}\left(\frac{r^4}{4}\right) = \frac{\rho_0 r^4}{4\epsilon_0 a^2}$ (radially out)

b) $q_{en} = 2\pi L\int_0^a \rho(r)\, r\,dr = \frac{2\pi L\rho_0}{a^2}\int_0^a r^3\,dr = \frac{2\pi L\rho_0}{a^2}\left(\frac{a^4}{4}\right) = \frac{\pi L\rho_0 a^2}{2}$

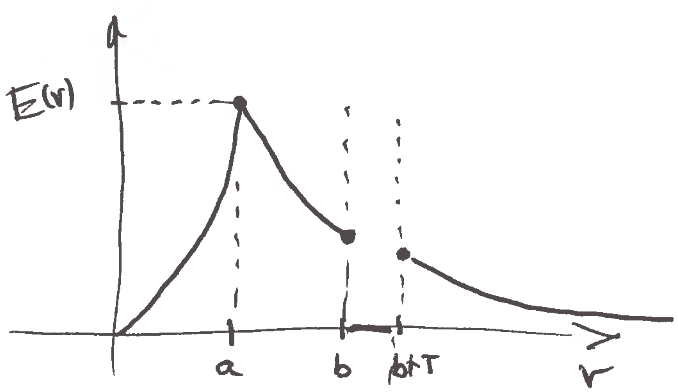
then $E(r) = q_{en}/2\pi r L\epsilon_0 = \left(\frac{\pi L\rho_0 a^2}{2}\right)\left(\frac{1}{2\pi r L\epsilon_0}\right)$

$= \frac{\rho_0 a^2}{4\epsilon_0 r}$ (r adially out)

c) inside a conductor $E = 0$ always!!

d) outside the conducting shell E-field will be the same as ~~that found~~ in part b).

e)



LO	P	F
15.2		
16.2		
7.5		
5.5		
18.1		
15.3		
16.3		
7.6		
5.6		
18.2		
19.2		
16.4		
18.3		
7.7		
5.7		
19.3		
3.6		