Physics 208 - Exam II

Spring 2018 (all sections) - March 5, 2018.

SOLUTIONS

Please fill out the information and read the instructions below, but **do not open the exam** until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 6 numbered (3 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. **Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!**
5. You will be allowed to use only non-programmable calculators on this exam.
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

Fill out the information below and sign to indicate your understanding of the above rules

Name:____________________ UIN:________________

(please print legibly)

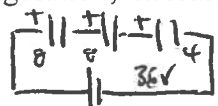
Signature:____________________ Section Number:____________

Instructor: Webb Saslow

(circle one)

A. You are given three capacitors of values 8.0 μF, 8.0 μF and 4.0 μF. They are connected in series and a voltage of 36 V is applied across the combination.

A.1) Sketch this configuration; indicate which is the + potential side.



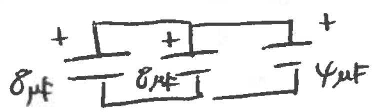
A.2) What is the charge on the 4.0 μF capacitor; indicate which is the + side?

$\frac{1}{C_T} = \frac{1}{8\mu F} + \frac{1}{8\mu F} + \frac{1}{4\mu F} = \frac{4}{8\mu F} \Rightarrow C_T = 2\mu F$ then $\boxed{Q_T \text{ on ALL} = (36V)(2\mu F) = 72\mu C}$

A.3) Using insulating gloves, we now disconnect the capacitors and re-connect them in parallel with all positively charged plates connected together and all the negatively charged plates connected together. Find the voltage across each capacitor in this new configuration.

NO BATTERY FOR PART (3)

$Q_T = 3Q = 216\mu C$



voltage across each capacitor is the same.

LO	P	F
6.1		
28.1		
30.1		
4.1		
5.1		
28.2		
30.2		

New charges after connection $\quad 2Q_8' + Q_4' = 216\mu C$

and $\quad \frac{Q_8'}{8\mu F} = \frac{Q_4'}{4\mu F}$ so $Q_4' = \frac{Q_8'}{2}$

then $2Q_8' + Q_4' = \frac{5}{2} Q_8' = 216\mu C \quad Q_8' = 86.4\mu C \quad Q_4' = 43.2\mu C$

$\boxed{V = Q/C = 43.2\mu C / 4\mu F = 10.8V}$

B. You are given a Wheatstone bridge with the three of its four resistances known. Note: when the bridge is balanced all the switches are closed and no current flows through the galvanometer in the circuit.

B.1) Within the circuit, what current flows from a-b-d?

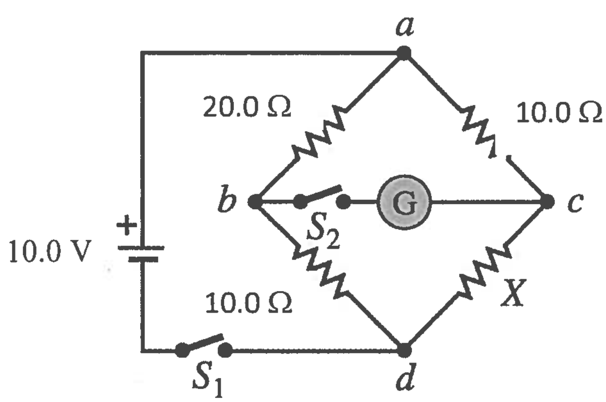
(1) $\boxed{I_{abd} = \frac{V}{R_T} = \frac{10V}{30\Omega} = .33A}$

B.2) What is the voltage (V_b-V_c) between points b and d?

(2) $V_b - V_c = 0$ since bridge is balanced

B.3) Within the circuit, what current flows from a-c-d?

B.4) What is the value of the unknown resistor **X**?



(3) current flow a-c-d

$V_b - V_a = V_c - V_a$

$20(1/3) = 10(I_{acd})$

$\boxed{I_{acd} = 2/3 A}$

LO	P	F
36.1		
38.1		
41.1		
36.2		
42.1		
36.3		
42.2		

(4) Voltage across X = Voltage across 10Ω

$10(\frac{1}{3}) = X(\frac{2}{3})$

$\boxed{X = 10(\frac{1}{3})\frac{3}{2} = 5\Omega}$

Problem I. Consider a capacitor consisting of two very thin coaxial cylinders with radii r_1 and r_2, and length L, as shown in the figure. Charges +/- Q are placed on the center conductor and the outer conductor, respectively.

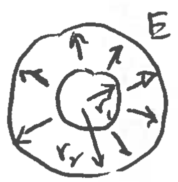
a) Indicate in the figure for $r_1 < r < r_2$ the direction of the electric field between the cylinders. Find the value of E(r) between r_1 and r_2. (Take these cylinders to be very long, so that end effects can be neglected. Please sketch the Gaussian surface that you plan to use.)

b) Calculate the potential difference $V_{r1} - V_{r2}$ between the inner and outer conductors.

c) Calculate the capacitance of this system.

d) If the region between the cylinders is filled with a dielectric with a dielectric constant κ, find the capacitance of this filled capacitor. Explain briefly.

a)



using Gauss's Law

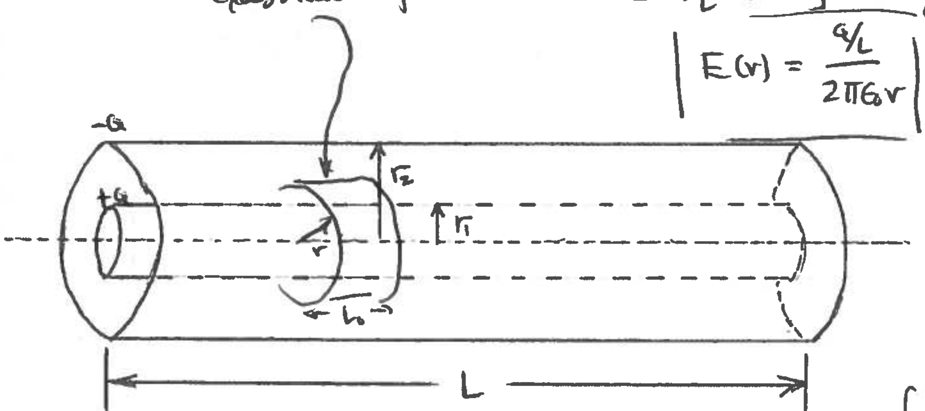
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{en}}{\epsilon_0} =$$

$$E(r)\left[2\pi r L_0\right] = \frac{\left(\frac{Q}{L}\right)L_0}{\epsilon_0}$$

$$\boxed{E(r) = \frac{Q/L}{2\pi\epsilon_0 r}}$$

	P	F
LO		
5.2		
13.1		
16.1		
18.1		
7.1		
24.1		
26.1		
29.1		
33.1		

Gaussian Surface



b) $V_{r_2} - V_{r_1} = -\int_{r_1}^{r_2} \vec{E}(r)\cdot d\vec{r} = -\frac{Q/L}{2\pi\epsilon_0}\int_{r_1}^{r_2}\frac{1}{r}\,dr = -\frac{Q/L}{2\pi\epsilon_0}\left[\ln(r_2) - \ln(r_1)\right]$

$$\boxed{= \frac{Q/L}{2\pi\epsilon_0}\ln\left(\frac{r_1}{r_2}\right) = V_{r_2} - V_{r_1}}$$

c) $$\boxed{C = \frac{Q}{|\Delta V|} = \frac{Q}{\frac{\frac{Q}{L}\ln(r_2/r_1)}{2\pi\epsilon_0}} = \frac{2\pi\epsilon_0 L}{\ln(r_2/r_1)}}$$

d) filling the space between r_1 & r_2 with dielectric increases the capacitance by the factor K

$$\boxed{C_{filled} = K\frac{2\pi\epsilon_0 L}{\ln(r_2/r_1)} = K\,C_{unfilled}}$$

Problem II. A voltage V_o is applied across a horizontal bar of length L, with a square cross-section, d on a side. A current I_o is flows through the bar. Answer the following in terms of the quantities given.

a) Indicate in the drawing the end of the bar that is at the highest potential.

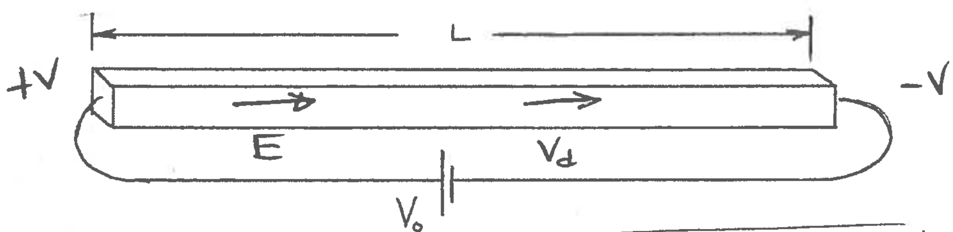
b) Find the electric field in the bar under these conditions. Give both magnitude and direction.

c) Find the resistivity of the bar.

d) Indicate the direction of the drift velocity of the charge carriers. Estimate its magnitude. The density of charge carriers in this material is n_o.

LO	P	F
22.1		
13.2		
25.1		
3.1		
35.1		
36.4		
13.3		
27.1		
36.5		

a)



b) $\boxed{E = \frac{\Delta V}{\Delta L} = \frac{V_o}{L}}$ pointing to the right.

c) $R = \frac{V}{I} = \frac{\rho L}{Area} = \frac{\rho L}{d^2}$ solving for ρ

$$\boxed{\rho = \frac{V d^2}{L I_o}}$$

d) Since $\vec{J} = n q \vec{v}_d = \frac{I_o}{d^2}$ (right)

$$\boxed{v_d = \left(\frac{I_o}{d^2}\right)\left(\frac{1}{n_o q}\right)}$$

q + to right

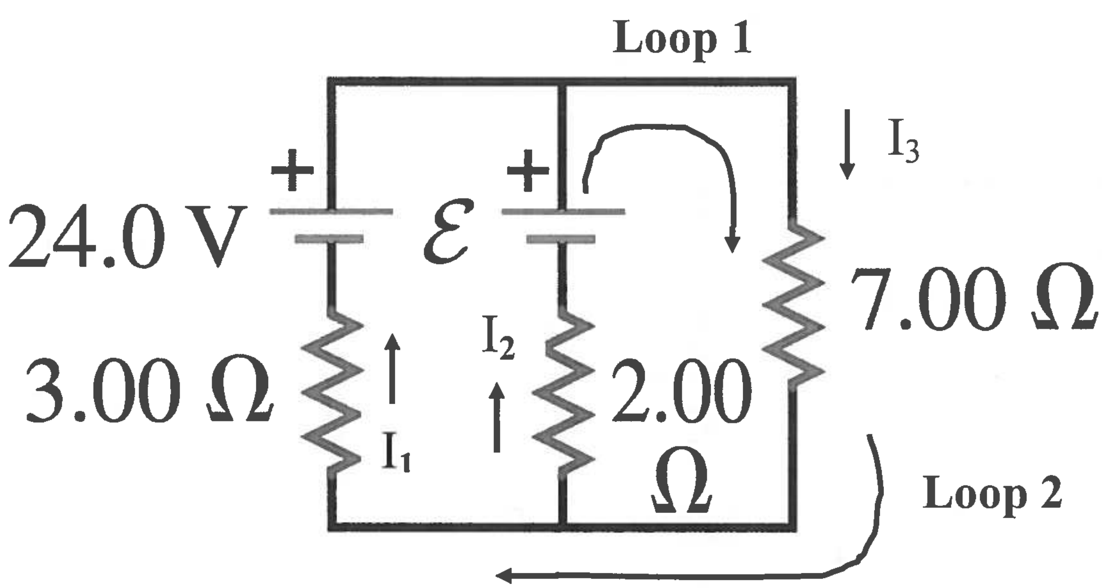
q − to left

Problem III. Consider the circuit below, containing two batteries with negligible internal resistance. In answering the questions below, **use the current directions and loop directions given in the figure.**

a) State the mathematical relationship between I_1, I_2 and I_3.

b) Using Kirchhoff's Laws, write down the loop equations for loops 1 and 2. (Be sure to label these equations, Loop1 and Loop2.)

c) If $I_3 = 1.80$ A find I_1, I_2 and $\mathcal{E}$.



a) $I_1 + I_2 = I_3$

b) Loop 1 $\quad \mathcal{E} - I_3 7 - I_2 2 = 0$

Loop 2 $\quad -I_3 7 - I_1 3 + 24 = 0$

Substituting for I_1 in Loop 2 gives

$-10 I_3 + I_2 3 + 24 = 0$

c) Solving for I_2 if $I_3 = 1.80$A use Loop 2

$-(1.8)10 + I_2 3 + 24 = 0$

$\boxed{I_2 = -2A}$

Then substituting I_2 and I_3 into Loop 1

$\mathcal{E} = I_3 7 + I_2 2 = (1.8)7 + (-2)(2)$

$\boxed{\mathcal{E} = 8.6V}$

LO	P	F
43.1		
38.2		
43.2		
38.3		
43.3		
4.2		
4.3		
4.4		

Problem IV. In the circuit below, the capacitor is initially uncharged and the switch is open. At t=0 the switch is closed. Take $\mathcal{E} = 6.0$ V, $R_1 = 6.0\ \Omega$, $R_2 = 3.0\ \Omega$ and $C_1 = 6.0\ \mu$F.

a) Find I, I_1, I_2 and Q_1 just after the switch is closed, and give your reasoning.

b) After the switch has been closed for a very long time, find I, I_1, I_2 and Q_1. Again give your reasoning.

c) On the graph below, sketch (you do not have to make a detailed graph) the charge on the capacitor as a function of time, for $t > 0$. Label the axes and be sure to indicate the range of charge and times being plotted.

d) Derive the functional form of $Q_1(t)$.

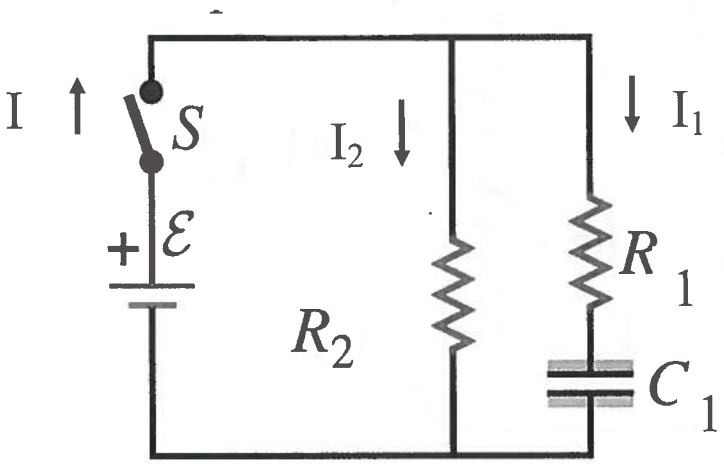
a) just after the switch is closed

$I_1 = \frac{\mathcal{E}}{R_1} = 3A$

$I_2 = \frac{\mathcal{E}}{R_2} = 2A$

$I = I_1 + I_2 = 5A$

$q = 0$ at start

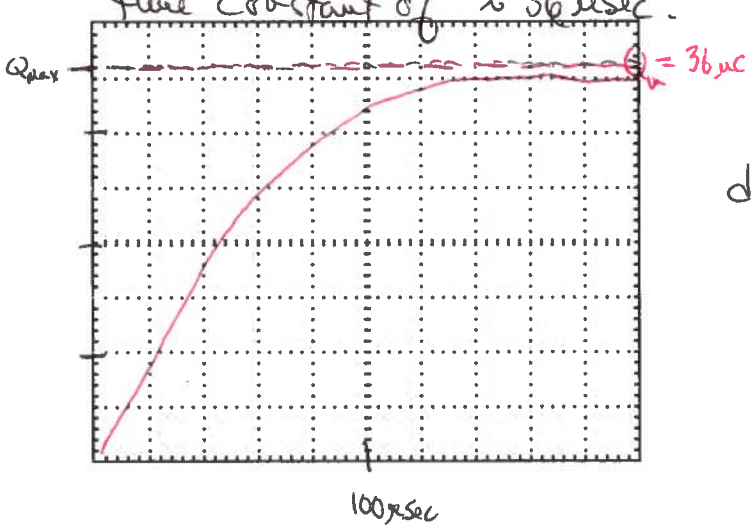


b) After switch closed for a long time, $I_1 = 0$, $I_2 = 2A$ and $I = 2A$

$Q = Q_{max} = VC = (I_2 R)C = (2A)(3\Omega)C = 36\mu C$

c) $RC = 6\Omega(6\mu F) = 36\mu sec$ ~~the time constant~~ for charging of capacitor.

Charge approaches Q_{max} asymptotically with time constant of $\sim 36\mu sec$.



LO	P	F
38.4		
38.5		
38.6		
41.2		
42.3		
45.1		
28.3		
41.3		
42.4		
45.2		
5.3		
45.3		
45.4		

d) $Q(t) = VC(1 - e^{-t/RC})$

$= 36\mu C\left(1 - e^{-t/36\mu sec}\right)$