Physics 208, Spring 2016 – Exam #3

SOLUTION

A

Name (Last, First): ______________________________

ID #: ________________

Section #: ________________

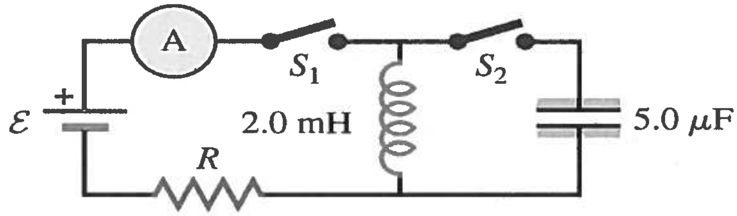
- You have 75 minutes to complete the exam.
- Formulae are provided on an attached sheet. You may NOT use any other formula sheet.
- You may use only a simple calculator: one without memory, or with a memory demonstrated to be cleared.
- When calculating numerical values, be sure to keep track of units. Results must include proper units.
- Be alert to the number of significant figures in the information given. Results must have the correct number of significant figures.
- If you are unable to solve a problem whose solution is needed in another problem, then assign a symbol for the solution of the first problem and use that symbol in solving the second problem.
- If you need additional space to answer a problem, use the back of the sheet it is written on.
- Show your work. Without supporting work, the answer alone is worth nothing.
- Mark your answers clearly by drawing boxes around them.
- Please write clearly. You may gain marks for a partially correct calculation if your work can be deciphered.

	MC Ver A	MC Ver B
MC 1.	E	B
MC 2.	E	A
MC 3.	C	D
MC 4.	D	C

1. (20 marks) In the circuit shown, switch S_1 has been closed for a long enough time so that the current reads a steady 5.50 A. Suddenly, switch S_2 is closed and S_1 is opened at the same instant.

a) What is the maximum charge that the capacitor will receive?

b) What is the current in the inductor at this time?



a) After $t=0$ when S_2 is closed and S_1 opened, energy is conserved in the LC circuit.

$$U_L(t=0) + U_C(t=0) = U_L(t) + U_C(t)$$ where $U_L(t) = \frac{1}{2} L I(t)^2$ & $U_C(t) = \frac{1}{2}\frac{Q(t)^2}{C}$

For maximum charge on capacitor,

$$U_L(t=0) + U_C(t=0) = U_L\left(t=\frac{T}{2}\right) + U_C\left(t=\frac{T}{2}\right)$$ where $T \equiv$ Period of oscillation

$$\Rightarrow \frac{1}{2} L I_{max}^2 + 0 = 0 + \frac{1}{2}\frac{Q_{max}^2}{C}$$

(because all the energy is in the inductor) (because all the energy is in the capacitor)

$$\Rightarrow \boxed{Q_{max} = \sqrt{LC}\, I_{max}}$$

$\longrightarrow 5.5 \times 10^{-4}$ C (Version A)

$\longrightarrow 2.5 \times 10^{-4}$ C (Version B)

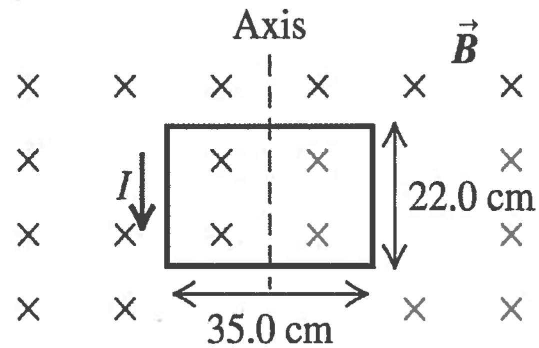
(b) When the capacitor has maximum charge, all the energy is in the inductor

$$\Rightarrow U_L = 0 \Rightarrow 0 = \frac{1}{2} L I^2 \Rightarrow \boxed{I_L = 0}$$

2. (20 marks) A rectangular coil of wire, 22.0 cm by 35.0 cm and carrying a current of 1.95 A, is oriented with the plane of its loop perpendicular to a uniform 3.50-T magnetic field.

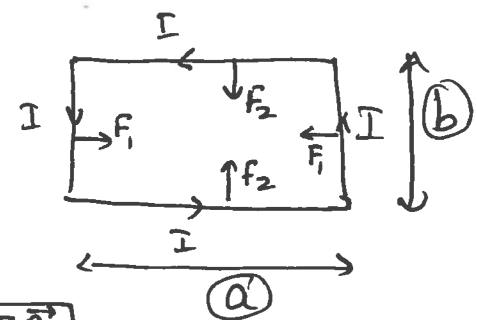
a) Calculate the net force and torque (magnitude and direction) that the magnetic field exerts on the coil.

b) The coil is rotated about a 25.0° angle about the axis shown, with the left side coming out of the plane of the figure and the right side going into the plane. Calculate the net force and torque (magnitude and direction) that the magnetic field now exerts on the coil. (*Hint:* To visualize this three-dimensional problem, make a careful drawing of the coil as viewed along the rotation axis.)



a.) Since B is uniform, each parallel segment experiences the same magnitude of force but in opposite directions using $\boxed{\vec{F} = I\vec{L} \times \vec{B}}$ (see figure)

$\Rightarrow \boxed{\Sigma \vec{F} = 0}$



$\Sigma \vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin\theta$ where $\boxed{\vec{\mu} = NI\vec{A}}$

Since $\boxed{\theta = \pi \text{ radians (NOT zero)}}$ $\boxed{\Sigma \vec{\tau} = 0}$

b) $\Sigma \vec{F} = 0$ again because of reason given in (a), each parallel segment will experience equal magnitude forces in opposite direction

$\Sigma \vec{\tau} = \vec{\mu} \times \vec{B} = (Iab)B \sin\theta$

Ver A : $\vec{\tau} = 1.95(0.35)(0.22)\ 3.5 \sin(155) = 0.222$ (N m)
OR $\sin(25)$

[direction: up along the axis OR counterclockwise viewed from above]

Ver B : 0.0664 (N-m)

[direction: down along the axis OR clockwise viewed from above]

3. (20 marks) A very long, straight wire with a circular cross section of radius $\boldsymbol{R}$ carries current $\boldsymbol{I}$. Assume that the current density is not constant across the cross section of the wire, but rather varies as $J = \beta r$, where β is a constant.

a) By the requirement that J integrated over the cross section of the wire gives the total current I, calculate the constant β in terms of I and R.

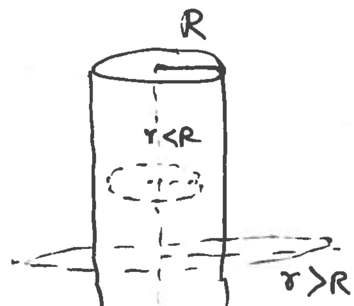
b) Use Ampere's Law to calculate the magnetic field $B(r)$ for (i) $r \le R$ and (ii) $r \ge R$. Express your answers in terms of I. .

a) $I = \int \vec{J} \cdot d\vec{a} = \int_0^R \underbrace{(\beta r)}_{\alpha \text{ (version B)}} 2\pi r \, dr = 2\pi\beta \int_0^R r^2 dr = \frac{2\pi}{3} \beta R^3$ (ver A)

$= \frac{2\pi}{3} \alpha R^3$ (ver B)

$$\Rightarrow \boxed{\beta \; (\text{or } \alpha) = \frac{3I}{2\pi R^3}}$$

b) For $r \le R$:



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$\Rightarrow B(2\pi r) = \mu_0 I_{enc} \Rightarrow \boxed{B(r) = \frac{\mu_0 I_{enc}}{2\pi r}}$

$I_{enc} = \int \vec{J} \cdot d\vec{a} = \frac{2\pi\beta r^3}{3}$ (using result from part a)

(Note: it is "r" here and NOT R. why?)

$\Rightarrow \boxed{I_{enc} = \frac{I r^3}{R^3}}$ (how?)

$$\Rightarrow \boxed{B(r) = \frac{\mu_0}{2\pi r}\left[\frac{I r^3}{R^3}\right] = \frac{\mu_0 I r^2}{2\pi R^3}}$$

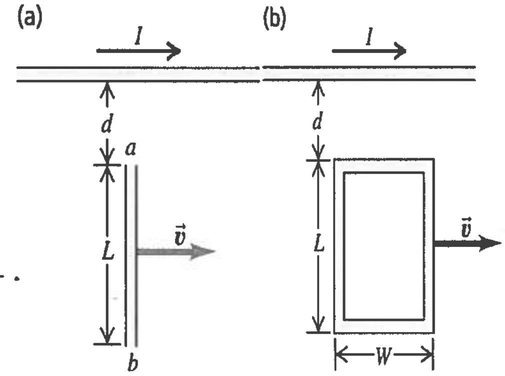
(c) For $r > R$ $\quad I_{enc} = I \quad \Rightarrow \boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$

4. (20 marks) A very long, straight wire shown in Figure (a) carries constant current $\boldsymbol{I}$. A metal bar with length $\boldsymbol{L}$ is moving with constant velocity $\vec{v}$, as shown in the figure. Point $\boldsymbol{a}$ is a distance $\boldsymbol{d}$ from the wire.

a) Calculate the emf induced in the bar (in figure (a)).

b) Which point, $\boldsymbol{a}$ or $\boldsymbol{b}$, is at a higher potential?

c) If the bar is replaced by a rectangular loop of resistance $\boldsymbol{R}$ (figure (b)), what is the magnitude of the current induced in the loop?



(a.) The I creates a B-field in all space.

[ver A: into page on the ~~rod~~ bar & the loop
ver B: out of page —"—]

Since it is a long (∞) wire, $\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$ (using Ampere's Law)

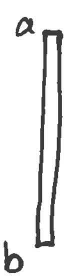
For the ~~rod~~ bar: The Emf is due to the motion of the conductor (motional Emf)

Under equilibrium: $\sum \vec{F}_{(\text{on free electrons})} = 0 \Rightarrow q\vec{E} + q(\vec{v}\times\vec{B}) = 0 \Rightarrow \boxed{\vec{E} = \ominus \vec{v}\times\vec{B}}$

$$\mathcal{E}mf = \Delta V = V(a) - V(b) = -\int_{d+L}^{d} \vec{E}\cdot d\vec{l}$$

$$= \oplus \int_{d+L}^{d} vB\sin 90\, dl = \int_{d+L}^{d} \frac{v\mu_0 I}{2\pi r} dr$$

$$= \frac{\mu_0 v I}{2\pi} \ln\left[\frac{d}{d+L}\right] \text{ (ver A) } \boxed{\text{and/OR}} \ \frac{\mu_0 v I}{2\pi} \ln\left[\frac{d+L}{d}\right] \text{ (ver B)}$$



$\boxed{\text{Ver A}}$ $\uparrow(\vec{v}\times\vec{B})$ $\downarrow\vec{E}$

$\boxed{\text{Ver B}}$ $\downarrow(\vec{v}\times\vec{B})$ $\uparrow\vec{E}$

(b.) Since $\vec{E}$ is pointing from $\vec{a}$ to b (ver A), "a" is at a higher potential (ver A)
(see figure above) "b" is at a higher potential (Ver B)

(c) $I_{\text{induced}} = \frac{\mathcal{E}mf}{R} = -\frac{1}{R}\frac{d\Phi_B}{dt}$ where $\Phi_B = \int \vec{B}\cdot d\vec{a}$

$\Phi_B \neq 0$ but since $\frac{d\Phi_B}{dt} = 0$, $\mathcal{E}mf = 0 \Rightarrow \boxed{I_{\text{induced}} = 0}$