

208 Mid-Term 1 POINTS TABLE

Multiple Choice (out of 20)	
Problem 1 (out of 20)	
Problem 2 (out of 20)	
Problem 3 (out of 20)	
Problem 4 (out of 20)	
TOTAL SCORE (out of 100)	

Multiple Choice:

Ver. A

1. e.)

2. b.)

3. a.)

4. b.), b.)

Ver. B

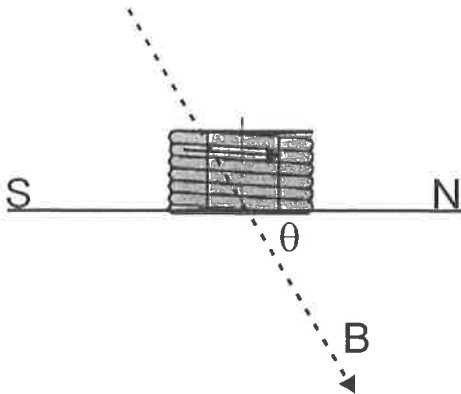
1. f?)

2. a.)

3. b.)

4. b.), a.)

1. (20 marks) A circular coil 28.0 cm in diameter and containing 18 loops lies flat on the ground. The Earth's magnetic field at this location has magnitude 5.80×10^{-5} T and points into the earth at an angle of $\theta = 61.0^\circ$ below a line pointing due north. If a 9.30-A counterclockwise (looking down on the coil) current passes through the coil, determine
- 16 a) the torque on the coil, and
4 b) which edge of the coil rises up, north, east, south or west.



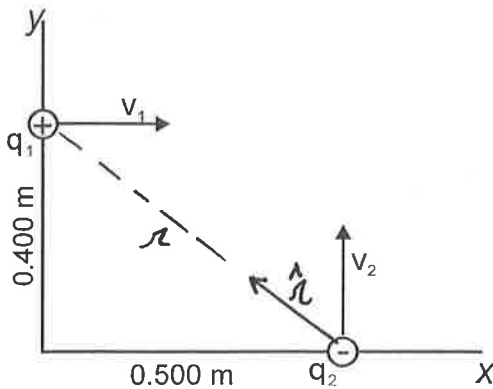
$$\begin{aligned}
 \text{a) } \vec{\tau} &= \vec{\mu} \times \vec{B} \\
 \vec{\mu} &= N I \vec{A} && \text{direction given by right hand rule - see figure} \\
 &= N I \pi r^2 \hat{j} \\
 \vec{\tau} &= N I \pi r^2 \hat{j} \times (B \cos 61^\circ \hat{i} - B \sin 61^\circ \hat{j}) \\
 &= -N I \pi r^2 B \cos 61^\circ \hat{k} \\
 &= -18 \times 9.30 \times \pi \times (.140)^2 \times 5.80 \times 10^{-5} \times \cos 61^\circ \hat{k} \\
 \vec{\tau} &= -2.90 \times 10^{-4} \text{ Nm } \hat{k} \quad (\text{ie into the page})
 \end{aligned}$$

b) With $\vec{\tau}$ into the page (pointing west), the right hand rule indicates that the south edge of the coil rises up.

x version: a) $\vec{\tau} = -15 \times 4.70 \times \pi \times (.210)^2 \times 5.80 \times 10^{-5} \times \cos 55^\circ \hat{k}$
 $\vec{\tau} = -3.25 \times 10^{-4} \text{ Nm } \hat{k}$ (ie into the page)
 b) same answer

2. (20 marks) A pair of point charges, $q_1 = 9.00\mu\text{C}$ and $q_2 = -6.00\mu\text{C}$, are moving as shown in the figure with speeds $v_1 = 6.50 \times 10^4 \text{ m/s}$ and $v_2 = 4.60 \times 10^4 \text{ m/s}$. When the charges are at the location shown in the figure, find:

- 8 a) the magnitude and direction of the magnetic field produced at the origin, and
 12 b) the magnitude and direction of the magnetic force that q_2 exerts on q_1 .



$$\begin{aligned}
 \vec{B} &= \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2} \\
 \vec{B}_{\text{origin}} &= \vec{B}_1 + \vec{B}_2 \\
 &= -\frac{\mu_0}{4\pi} \left[\frac{q_1 v_1}{r_1^2} + \frac{q_2 v_2}{r_2^2} \right] \hat{k} \\
 &= -10^{-7} \left[\frac{9.00 \times 10^{-6} \times 6.50 \times 10^4}{(0.400)^2} + \frac{6.00 \times 10^{-6} \times 4.60 \times 10^4}{(0.500)^2} \right] \hat{k} \\
 \vec{B}_0 &= -4.76 \times 10^{-7} \text{ T } \hat{k}
 \end{aligned}$$

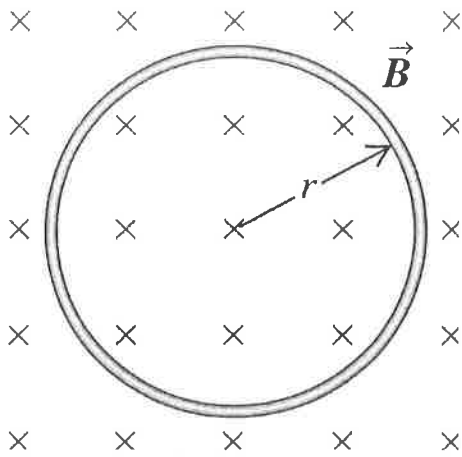
b) See diagram

$$\begin{aligned}
 \vec{F} &= q_1 v_1 B_2 \hat{j} = q_1 v_1 \left(\frac{\mu_0}{4\pi} \frac{q_2 v_2 \sin \theta}{r^2} \right) \hat{j} \quad \text{where } \sin \theta = \frac{0.500}{r} \\
 &= 9.00 \times 10^{-6} \times 6.50 \times 10^4 \times 10^{-7} \times \frac{6.00 \times 10^{-6} \times 4.60 \times 10^4 \times 0.500}{(0.400^2 + 0.500^2)^{3/2}} \hat{j}
 \end{aligned}$$

$$\vec{F} = 3.08 \times 10^{-8} \text{ N } \hat{j}$$

3. (20 marks) A circular coil containing 50 loops of wire with radius $r = 35$ cm and resistance $R = 0.460 \Omega$ is in a region of spatially uniform magnetic field directed into the plane of the figure. At $t = 0$, $B = 0$. The magnetic field then begins increasing, with $B(t) = (0.560 \text{ T/s}^3) t^3$.

- 16 a) What is the current in the loop (magnitude and direction) at the instant when $B = 1.56$ T?
 4 b) What is the power being dissipated in the loop at that instant?



$$\begin{aligned} \text{a) } \mathcal{E} &= -N \frac{d\Phi_B}{dt} = -N \frac{d(BA)}{dt} = -NA \frac{dB}{dt} \\ &= -NA (3t^2 \times 0.560) \\ &= -50 \times \pi (0.035)^2 \times 3 \times 0.560 t^2 \\ &= -0.3233 t^2 \end{aligned}$$

$$\begin{aligned} \text{but } B &= 1.56 = 0.560 t^3 \Rightarrow t^3 = 2.786 \\ \text{or } t &= 1.407 \text{ s} \end{aligned}$$

$$\therefore |\mathcal{E}| = +0.3233 \times 1.407^2 = 0.6401 \text{ V}$$

$$\boxed{I = \frac{\mathcal{E}}{R} = \frac{0.6401 \text{ V}}{0.460 \Omega} = 1.39 \text{ A counter-clockwise}}$$

$$\text{b) } P = \mathcal{E}I = 0.6401 \times 1.391 = \boxed{0.891 \text{ W}}$$

4. (20 marks) A 1200-pF capacitor is charged to 100 V and then quickly connected to an inductor. The frequency of the subsequent oscillation is 720 kHz. Determine

- 6 a) the inductance L of the inductor
- 7 b) the peak value of the current in the inductor, and
- 7 c) the maximum energy stored in its magnetic field.

$$a) \omega = 2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow LC = \frac{1}{(2\pi f)^2}$$

$$\therefore L = \frac{1}{C(2\pi f)^2} = \frac{1}{1200 \times 10^{-12} \text{ F} (2\pi \times 720 \times 10^3 \text{ Hz})^2}$$

$$L = 40.7 \mu\text{H}$$

b) Initially, all energy is in electric field of the capacitor

$$U_E = \frac{1}{2} CV^2 = \frac{1}{2} \times 1200 \times 10^{-12} \text{ F} (100 \text{ V})^2 = 6.00 \mu\text{J}$$

The peak value of current occurs when all energy is stored in magnetic field of inductor

$$\therefore U_B = \frac{1}{2} LI^2 = U_E$$

$$\therefore I^2 = \frac{2U_E}{L} = \frac{2 \times 6.00 \times 10^{-6} \text{ J}}{40.7 \times 10^{-6} \text{ H}} = 0.2948 \text{ A}^2$$

$$I = 543 \text{ mA}$$

c) Maximum energy is U_B as defined above

$$U_B = U_E = 6.00 \mu\text{J}$$

x version: a) $L = \frac{1}{800 \times 10^{-12} \text{ F} (2\pi \times 940 \times 10^3 \text{ Hz})^2}$

$$L = 35.8 \mu\text{H}$$

b) $U_E = \frac{1}{2} \times 800 \times 10^{-12} \text{ F} \times (110 \text{ V})^2 = 4.84 \mu\text{J}$

$$I = \sqrt{\frac{2 \times 4.84 \times 10^{-6} \text{ J}}{35.8 \times 10^{-6} \text{ H}}}$$

$$I = 520 \text{ mA}$$

c) $U_B = U_E = 4.84 \mu\text{J}$