Physics 208 - Exam III

Spring 2018 (all sections) - April 9, 2018.

SOLUTION

Please fill out the information and read the instructions below, but
do not open the exam until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 8 numbered (4 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. **Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!**
5. You will be allowed to use only non-programmable calculators on this exam.
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

Fill out the information below and sign to indicate your understanding of the above rules

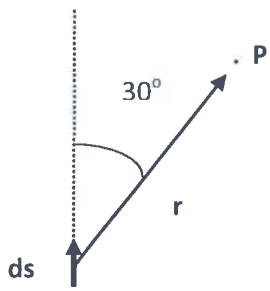
Name:______________________________ UIN:____________________

(please print legibly)

Signature:____________________________ Section Number:______________

Instructor: **Webb** **Saslow**
(circle one)

A. Consider an element **ds** of a current carrying wire with I = 4.0 A and |**ds**| = 0.10 mm. At point P, a distance of 40.0 mm from **ds** as shown in the figure, find: **i)** the direction of the magnetic field produced by this current element and **ii)** the magnitude of the magnetic field **dB** produced by this current element.



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{\ell} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{(4A)(10^{-4} m)}{(4\times10^{-2} m)^2} \sin 30° \quad \text{INTO PAGE AT P}$$

$$= 1.25\times10^{-8}\ T$$

LO	P	F
2.1		
3.1		
5.1		
52.1		

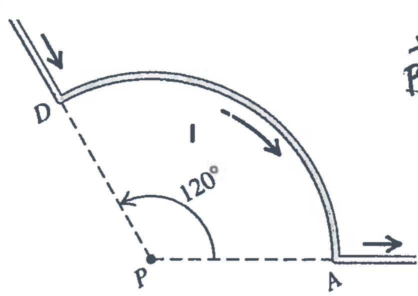
B. Consider the current carrying wire in the figure below, where the circular arc portion has radius R and subtends 120°. **i)** Find the direction of the magnetic field that this wire produces at the point P; and find the magnitude of the magnetic field at P due to **ii)** the wire segment coming in from infinity at 120° to point D, **iii)** the circular arc, **iv)** the wire segment going from A to infinity. since $\hat{r} \parallel d\vec{\ell}$

$$\vec{B}_{at\,P} = \frac{\mu_0}{4\pi} \int \frac{i\, d\vec{\ell} \times \hat{r}}{r^2} = \cancel{\int_{\infty}^{D}}^{\,0} + \int_{D}^{A} + \cancel{\int_{A}^{\infty}}^{\,0} \quad \text{since } \hat{r} \parallel d\vec{\ell}$$

$$= \frac{\mu_0}{4\pi} \int_D^A \frac{i\, d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I\left(\frac{2\pi R}{3}\right)}{R^2} \quad \text{INTO PAGE}$$

LO	P	F
2.2		
3.2		
5.2		
52.2		

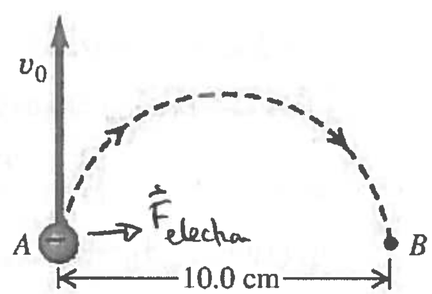
$$\vec{B}_{at\,P} = \frac{\mu_0 I}{6R} \quad \text{INTO PAGE}$$



C. Consider the following configurations and for each answer the question that has been posed for that figure.

i. An electron moves upward into a region of magnetic field, and then deflects rightward in a semicircle. See figure 1. Give the direction of B.

Figure 1)

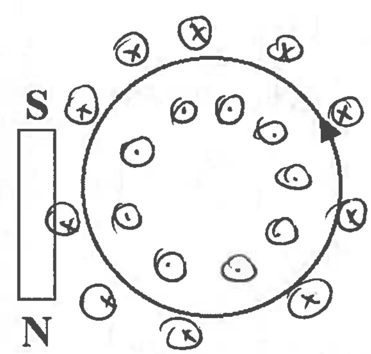


$\vec{F}_{electron} = q\vec{v} \times \vec{B}$ to the right

B must be into page for a negatively charged particle to bend to right.

ii. A wire loop carries current counterclockwise and initially is in the same plane as a magnet. If the magnet's center of mass cannot move, but it is able to rotate in any direction about its center of mass, how will the magnet rotate?

Figure 2)

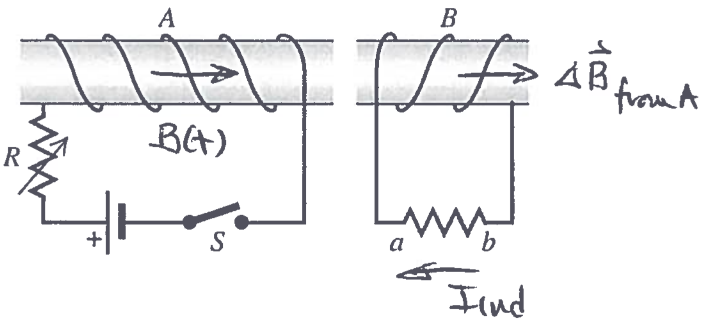


Bar magnet will align with the $\vec{B}$-field of the loop outside with N pole pointing into the page and S pole pointing out.

LO	P	F
2.3		
6.1		
46.1		
50.1		
52.3		
3.3		

iii. In figure 3), at t = 0 the switch S for circuit A closes. What is the direction of the change in field **B** due to circuit A, as seen by circuit B? On figure 3 give the direction of the induced current I_B between a and b.

Figure 3)

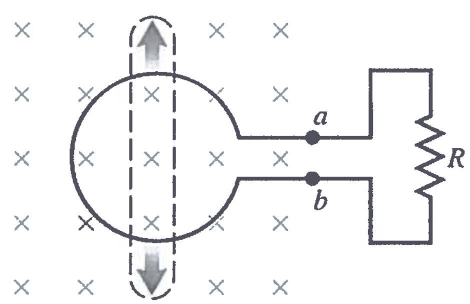


Flux increase in $\vec{B}$ due to changing current in A.

Induced current will flow to counteract this increase with $\vec{B}_{ind}$ pointing to the left. and I_{ind} from b to a

iv. The circular loop in figure 4) is in a uniform field, $\mathbf{B_o}$. At t = 0 it is stretched (dashes). On figure 4 give the direction of the induced current between a and b.

Figure 4)



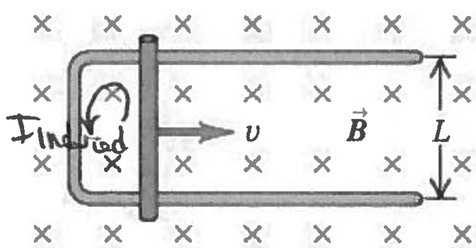
As the loop is deformed its area decreases, so flux decreases.

LO	P	F
47.1		
55.1		
56.1		
57.1		
58.1		
60.1		
47.2		
56.2		
57.2		
58.2		
60.2		

The induced current will circulate to replace the decreasing flux with $\vec{B}_{INDUCED}$ into the page. So $I_{induced}$ must flow Clockwise.

Problem I. A slide wire of mass m and large resistance R, with an initial position x_o and velocity v_o, is on a rectangular loop of width L in a magnetic field B as shown in the figure. Answer the following in terms of the quantities given, x_o, v_o, R, m, L, and B.

a) Find the induced emf associated with the motion of the slide wire.
b) Find the induced current I in the wire, magnitude and direction.
c) Find the force F on the slide wire required to keep the wire moving at constant velocity, including direction.
d) Find the power loss/gain associated with F during this motion.
e) Find the rate of Joule heating during this motion.



a) $\mathcal{E}mf_{\text{induced}} = -\dfrac{d\phi_B}{dt}$

$= -\dfrac{d}{dt}(BLX) = -BL\dfrac{dx}{dt}$

$= -BLv_0$

b) $|I_{\text{induced}}| = \dfrac{|\mathcal{E}mf|}{R} = \dfrac{BLv_0}{R}$

Counter clockwise

c) $d\vec{F}_{\text{WIRE}} = I\, d\vec{l} \times \vec{B} = \left(\dfrac{BLv_0}{R}\right) LB$ (to the left)

so we must apply an equal force to the right ~~for the system to move~~ with constant speed.

LO	P	F
7.1		
57.3		
37.1		
58.3		
2.4		
49.1		
40.1		
40.2		
3.4		

d) Power Supplied $= \vec{F} \cdot \vec{v} = -\dfrac{v_0^2 B^2 L^2}{R}$

e) Joule heating of R

$P = I^2 R = \left(\dfrac{BLv_0}{R}\right)^2 R = \dfrac{B^2 L^2 v_0^2}{R}$

Problem II. A coaxial cable has two parts. An inner solid conductor of radius of a carries a current I into the page, and uniformly distributed over its cross section. Supported by insulating disks is a co-axial conducting tube of inner radius b and outer radius c that carries a current of $4I$ out of the page and uniformly distributed over its cross section.

a) On the drawing indicate how the net magnetic field circulates in the regions:

i. $r < a$,

ii. $a < r < b$, and

iii. $r > c$; ignore $b<r<c$.

b) Find the magnetic field as a function of radius r for the following regions,

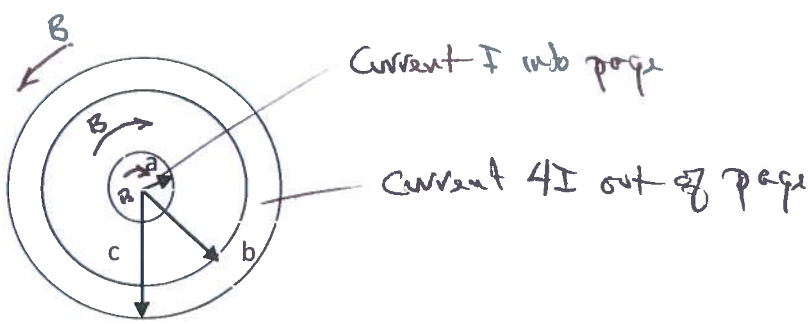
i. $r < a$,

ii. $a < r < b$, and

iii. $r > c$; ignore $b<r<c$. (For full credit show all the steps in your calculation.)

c) Sketch B as a function of r; interpolate for the region b<r<c.

a)



LO	P	F
54.1		
55.2		
7.2		
54.2		
54.3		
54.4		
3.5		
54.7		

b) Using Ampere's Law for

$r<a$

$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I = \mu_0 I\left(\frac{r^2}{a^2}\right)$$

$$B(r) = \left(\frac{\mu_0 I}{2\pi a^2}\right) r \quad (\text{CW})$$

$a<r<b$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

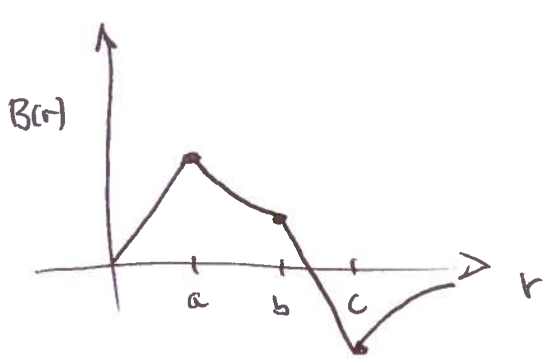
$$B(r) = \frac{\mu_0 I}{2\pi r} \quad (\text{CW})$$

$r>b$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 (I-4I)$$

$$B(r) = \frac{\mu_0 3I}{2\pi r} \quad (\text{CCW})$$

c)



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Problem III. An inductor with inductance $L = 5.0$ mH and negligible resistance is connected to a battery of EMF 12 V and negligible internal resistance, a switch, and two resistors, $R_1 = 60\ \Omega$ and $R_2 = 30\ \Omega$, as shown.

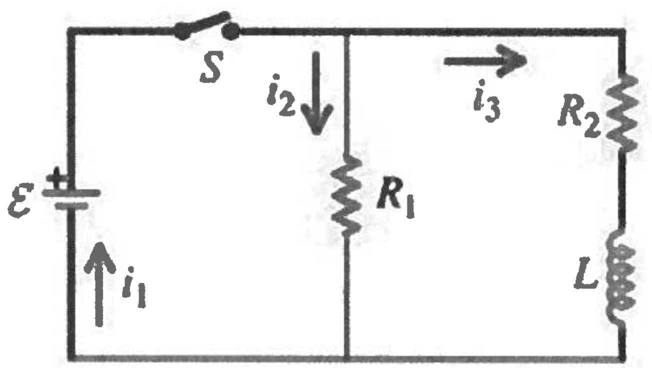
a) At t = 0 the switch is closed. Find the currents i_1, i_2 and i_3 immediately after the switch is closed.

b) After the switch has been closed for a long time, find the currents i_1, i_2 and i_3.

c) What is the energy stored in the inductor at this time?

d) If now we once again open the switch, find the currents i_1, i_2 and i_3 at that time and indicate their directions.

e) How long does it take for the current in the circuit to decrease by a factor of 3 after the switch is opened?



a) $i_1 = \frac{\mathcal{E}}{R_1} = \frac{12}{60}$

$i_2 = i_1$

$i_3 = 0$

b) $i_1 = i_2 + i_3$

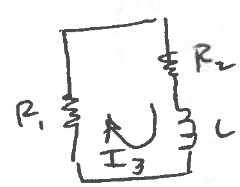
$i_2 = \frac{\mathcal{E}}{R_1} = \frac{12}{60}$

$i_3 = \frac{\mathcal{E}}{R_2} = \frac{12}{30}$

c) $U_{IND} = \frac{1}{2} L I^2 = \frac{1}{2}(5\times10^{-3})\left(\frac{12}{30}\right)^2$

d) $i_1 = 0$; $i_2 = -i_3$; $i_3 = \frac{12}{30}$

e) The new circuit



$-(R_1+R_2)I - L\frac{dI}{dt} = 0$

solving for $I(t)$

$$I(t) = i_3\, e^{-\frac{(R_1+R_2)}{L}t}$$

for $I(t) = \frac{i_3}{3} = i_3 e^{-\frac{(R_1+R_2)}{L}t}$

$-\ln 3 = -\frac{(R_1+R_2)}{L}t$ solving for $t = \ln 3 \left(\frac{L}{R_1+R_2}\right)$

LO	P	F
42.1		
65.1		
42.2		
65.2		
64.1		
42.3		
65.3		
7.3		
43.1		
65.4		
3.6		

Problem IV. A rectangular wire loop is slipped between two turns of a long solenoid with a square cross section of side 10.0 cm. The loop has dimensions $b = 12.0$ cm and $l = 2.0$ cm, and it protrudes by $x = 6.0$ cm. When the current in the solenoid is $I_{sol} = 6.0$ A clockwise the field within the solenoid is $B_{sol} = 0.06$ T. Answer the following:

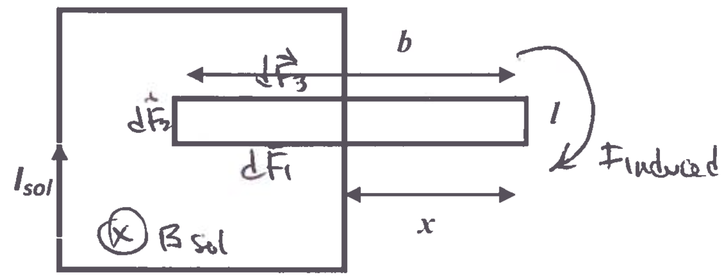
a) Indicate on the figure the direction of B_{sol}.

b) If I_{sol} starts to change at the rate $dI_{sol}/dt = -500$ A/s, find the rate of change of the magnetic flux through the rectangular loop.

c) If the loop has resistance 25 Ω, find the current induced in the loop and indicate its direction on the figure.

d) Find the net force on the rectangular wire loop, including its direction, for the current found in part c).

e) From b), find the mutual inductance.



a) B_{sol} is into the page

b) $\frac{d\phi_B}{dt} = \frac{d}{dt}\int \vec{B}\cdot d\vec{A} = \frac{d}{dt}\left((\mu_0 n\, i_{sol})\, l\,(b-x)\right)$

$= \mu_0 n l (b-x) \frac{di_{sol}}{dt}$

where $B = \mu_0 n\, i_{sol} = 0.06\text{T} = (4\pi\times10^{-7})\, n\, 6\text{A}$

Solving for $n = \frac{0.06}{(4\pi\times10^{-7})(6)}$

	P	F
LO		
54.5		
54.6		
56.3		
57.4		
58.4		
60.3		
46.2		
49.2		
3.7		
62.1		

c) $\mathcal{E}_{mf\,induced} = -\frac{d\phi_B}{dt} = -\mu_0 n l (b-x)\frac{di_{sol}}{dt}$

$i_{induced} = \frac{\mathcal{E}_{mf}}{R} = \frac{\mu_0 n l (b-x)\frac{di}{dt}}{R}$ (in CW direction)

d) $\vec{F}_{total} = d\vec{F}_1 + d\vec{F}_2 + d\vec{F}_3$ (see figure)

$= d\vec{F}_2 = I_{induced}\, l\, B(t)$ (to left)

e) $\mathcal{E}_{mf\,induced} = -M\frac{dI}{dt}$ solving for $M = \left|\frac{\mathcal{E}_{mf}}{dI/dt}\right| = \mu_0 n l (b-x)$