SOLUTIONS

Physics 207 - Exam II

Fall 2019 (207-210, 543-566; 579-584) October 14, 2019.

Please fill out the information and read the instructions below, but
do not open the exam until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 6 numbered (3 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!
5. **You will be allowed to use only non-programmable calculators on this exam.**
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be quite distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

Fill out the information below and sign to indicate your understanding of the above rules

Name:____ UIN:__________________
(printed legibly)

Signature:_________ Section Number: _________

Instructor: Webb Kocharovskaya Saslow Eusebi
(circle one)

A. A parallel plate capacitor with an air gap has electrical energy 1.8×10^{-5} J when connected to a 3.0 V battery. This is the initial configuration mentioned in the questions below.

a) Find the charge on this capacitor.

b) What is the value of this capacitance?

The capacitor is now disconnected from the battery and a slab of dielectric with dielectric constant κ=4 and nearly the same thickness as the capacitor is then slid into space between the two plates of the capacitor. This is the final configuration referenced below.

c) Find the final charges on the plates of this capacitor.

d) Find the final voltage difference between the plates of the capacitor.

e) Find the energy stored in the capacitor after the dielectric is introduced.

a) $U = \frac{1}{2} QV = 1.8 \times 10^{-5}$ J

$Q = \frac{2(1.8 \times 10^{-5} \text{ J})}{3 V} = 12 \mu C$

LO	P	F
3.1		
28.1		
28.2		
31.1		
31.2		
34.1		
34.2		

b) $C = \frac{Q}{V} = \frac{12 \mu C}{3 V} = 4 \mu F$

c) After being disconnected from the battery, the charge Q remains the same.

$Q_{after} = 12 \mu C$

d) $V_{New} = V_{old}/K = \frac{3V}{4} = 0.75V$

e) $U_{New} = \frac{1}{2} Q V_{New} = \frac{1}{2}(12 \mu C)(.75V)$

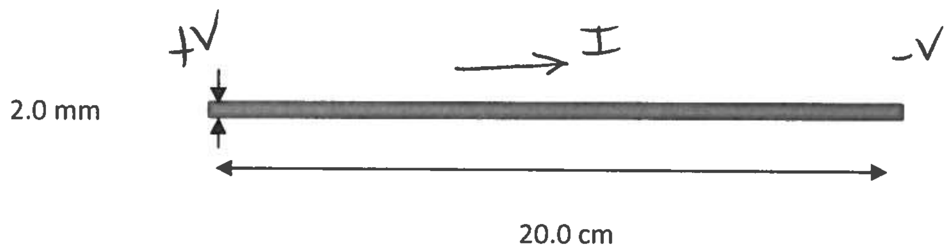
$= 4.5 \times 10^{-6}$ J

B. Consider a conducting rod 20 cm long with radius 2 mm. It carries 4 A when a voltage difference of 0.5 V **(high voltage to left)** is placed across its ends.

a) Find the resistivity of the rod.

b) In which direction is the current flowing in this rod. Indicate this direction in the figure.

c) Find the electric field E within the rod, make sure you indicate its direction.



LO	P	F
3.2		
35.1		
36.1		
36.2		
~~37.1~~		

a) $R = \frac{\Delta V}{I} = \frac{0.5V}{4A} = 0.125\,\Omega$

Since $R = \frac{\rho L}{A} = 0.125\,\Omega$

Solving for $\rho = \frac{0.125\,\Omega\,(\pi(2\times10^{-3})^2)}{.2m} = 7.8\times10^{-6}\,\Omega m$

b) See figure

c) $|E| = ?$ Since $-\oint \vec{E}\cdot d\vec{l} = \Delta V$

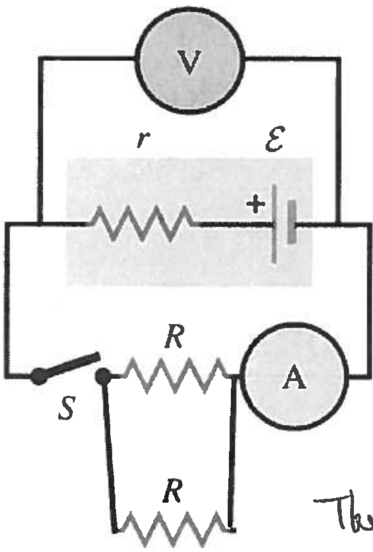
$= \left|\frac{-\Delta V}{\Delta L}\right| = \left|\frac{-0.5V}{0.2m}\right| = 2.5\,V/m$

direction of E is in the direction of the current above.

C. The circuit below contains a non-ideal battery with an internal resistance r and an emf = $\mathcal{E}$. The meters shown in the circuit are "ideal" volt and ammeters. **Answer the following in terms of the quantities given:**

a) **Before the switch is closed**, what are the "readings" on the voltmeter and ammeter?

b) **After the switch is closed**, what are the "readings" on the voltmeter and ammeter?



a) Before switch is closed

No current is flowing in the circuit

so Ammeter reads 0 A.

The voltmeter is across the battery

so voltmeter reads $\mathcal{E}$.

b) After the switch is closed

The circuit will have a resistance

of $R_{TOTAL} = r + \frac{R}{2}$ [from 2R in parallel]

So the current in the loop will be

$I = \mathcal{E}/(r + R/2)$ and this is the

reading on the ammeter.

The voltmeter across the battery will read

the terminal voltage when the current flows

$$V = \mathcal{E} - Ir = \mathcal{E} - \left(\frac{\mathcal{E}}{r + R/2}\right)$$

$$= \mathcal{E}\left(1 - \left(\frac{r}{r + R/2}\right)\right)$$

LO	P	F
3.3		
36.3		
38.1		
39.1		
41.1		
41.2		
44.1		
44.2		

Problem 1.

Consider the three capacitors shown in the circuit below. $C_1 = 9\ \mu F$ and $C_2 = 21\mu F$ are in parallel, and $C_3 = 15\mu F$ is in series with them. $V_a = 3$ V and $V_b = -12$ V.

a) Find the charge on each of the capacitors in this circuit.

b) Find the voltage difference across each capacitor.

c) Find V_d.

d) What is the **total** energy stored in these three capacitors in this configuration?

a) $C_{Eq}^{1,2} = 9 + 21 = 30\ \mu F$ $\quad C_{Eq}^{12,3} = \left(\frac{1}{30} + \frac{1}{15}\right)^{-1} = 10\ \mu F$

$q_3 = C_{Eq}^{12,3} \cdot 15V = 150\ \mu C$

$\Delta V_{C_3} = \frac{q_3}{C_3} = \frac{150\mu C}{15\mu F} = 10V \Rightarrow \Delta V_{C_1} = \Delta V_{C_2} = 5V.$

$\Rightarrow q_2 = C_2 5V = 21\mu F \cdot 5 = 105\ \mu C$, $\quad q_1 = C_1 5V = 45\ \mu C$

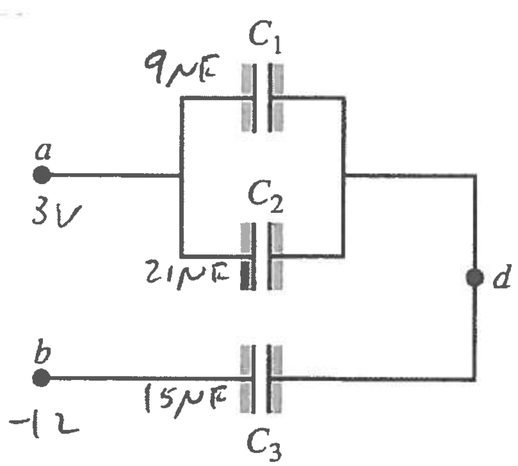
b) $\Delta V_{C_1} = \Delta V_{C_2} = 5V$

$\Delta V_{C_3} = 10V$

c) $V_d = -12 + 10V = -2V$

d) $\frac{1}{2} \frac{Q^2}{C_{eq}^{123}} = \frac{1}{2} \frac{(150\mu C)^2}{10\mu F} = 1125\ \mu J.$

$\Delta V = 15V$



LO	P	F
3.4		
28.3		
28.4		
30.1		
30.2		
31.3		
43.1		
28.5		
28.6		

Problem II.

The capacitor in the circuit is initially uncharged. At t = 0 the switch is closed.

a) Find the current flowing through each of the resistors in the circuit just after the switch is closed.

b) What will be the current flowing through each of these resistors after the switch has been **closed a very long time**?

c) What is the characteristic time constant of this circuit?

d) **After waiting a long time after the switch is closed**, what will be the charge on the capacitor?

e) **After waiting a long time the switch is opened**. Find the current flowing in each resistor at the instant the switch is opened.

a) at $t=0$ $\quad R_{TOT} = \left(6\Omega + \left(\frac{1}{8}+\frac{1}{4}\right)^{-1}\right) = 8.67\Omega$

$I_{TOTAL} = 40V/8.67\Omega = 4.61\,A = I_1 + I_2$

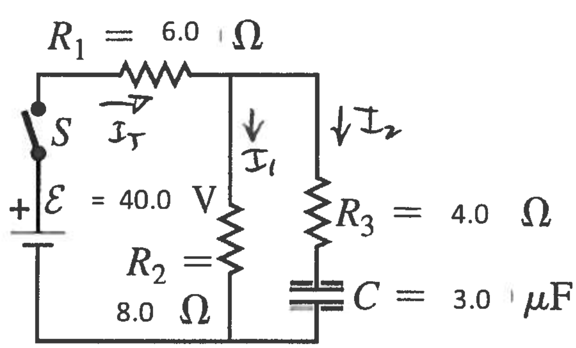
$I_1 = 4.61\left[\frac{4}{4+8}\right] = 1.53A$; $I_2 = 3.07A$

b) at $t = \infty$ $\quad I_{TOTAL} = I_1 = 40V/14\Omega = 2.86\,A$

c) this part was not graded since the circuit was not a simple series RC circuit. The correct answer is $\tau = C\left[R_1 + R_3\left(1+\frac{R_1}{R_2}\right)\right]$ $\cong 39\,\mu sec$

d) $Q = VC = (I_1R_2)C = 68.6\,\mu C$

e) $I_{R_2} = I_{R_3} = \frac{Q/C}{R_2+R_3} = 1.91\,A$



LO	P	F
3.5		
36.4		
36.5		
36.6		
38.2		
41.3		
41.4		
~~41.5~~		
45.1		
~~45.2~~		
45.3		
45.4		

Problem III.

Given the circuit below, answer the following:

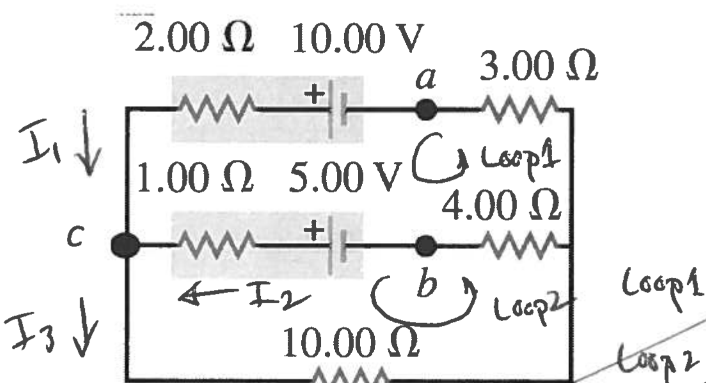
a) Clearly label the currents flowing in each branch of the circuit.

b) Write an expression for the current flowing into and out of the junction marked *c* using your currents as labeled in part a).

c) Find the currents flowing in each branch of the circuit.

d) What is the voltage between points a and b in the circuit, $V_a - V_b$?

e) Find the power that each battery supplies to the circuit.



a) See figure

b) $I_1 + I_2 = I_3$

c)

Loop1 $10V - 5V - 5I_1 + 5I_2 = 0$

Loop2 $5V - 5I_2 - 10I_3 = 0$

Solving for I_1, I_2 & I_3

$1 - I_1 + I_2 = 0$

$1 - I_2 - 2I_3 = 0$

Substitute for I_3 to get $1 - 3I_2 - 2I_1 = 0$

Then solve for $I_2 = -1/5$, $I_1 = 4/5$ & $I_3 = 3/5$

d) $V_a - V_b = -\left[10V - \frac{4}{5}2\Omega - \frac{1}{5}1\Omega - 5V\right]$

$= -\left[\frac{16}{5}V\right]$

e) Power $= IV_{\text{terminal battery}}$

$P_{10V} = \left(\frac{4}{5}\right)\left(10V - 8/5\,V\right) = 6.72$

$P_{5V} = \left(-\frac{1}{5}\right)\left(5V + 1/5\,V\right) = -1.04$ (the battery is being charged)

LO	P	F
3.6		
4.1		
4.2		
4.3		
38.3		
~~39.2~~		
~~39.3~~		
40.1		
40.2		
42.1		
43.2		
43.3		
43.4		