SOLUTION

Physics 207 - Exam I

Fall 2019 (207-210, 543-566; 579-584) September 23, 2019.

Please fill out the information and read the instructions below, but
do not open the exam until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 6 numbered (3 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!
5. **You will be allowed to use only non-programmable calculators on this exam.**
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be quite distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

Fill out the information below and sign to indicate your understanding of the above rules

Name:____________________________________ UIN:________________
(printed legibly)

Signature:_________ Section Number: _________

Instructor: Webb Kocharovskaya Saslow Eusebi
(circle one)

A. Consider an insulating rod of length $2L$ with a charge per unit length, λ, aligned with the x-axis with one end at $x = 0$ and the other a $x = 2L$. You are also given that $\lambda(x)$ has the following dependence on x, the position along the length of the line segment,

$\lambda(x) = C_o (x/L^2)$, where C_o is a constant. In terms of the constants given, find the total charge of this line segment.

$$q_{total} = \int dq = \int_0^{2L} \lambda(x)dx = \int C_0\left(\frac{x}{L^2}\right)dx = \left(\frac{C_0}{L^2}\right)\frac{1}{2}x^2\Big|_0^{2L}$$

$$= \left(\frac{C_0}{2L^2}\right)(2L)^2 = 2C_0$$

LO	P	F
3.1		
5.1		
7.1		

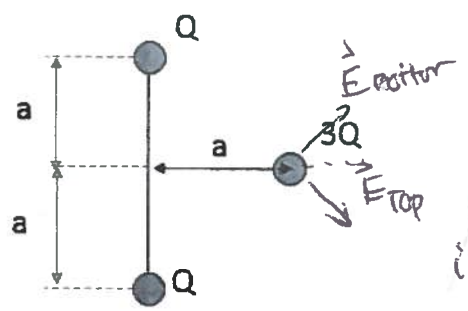
B. An electrical "sling shot" is made out of two charged particles each with a charge $+Q$ that are held together by a rigid bar of length $2a$. The bar is fixed in space and cannot move. A third charge, $3Q$, is initially placed a distance of a from the bar as shown.

i) Find the electric field due to the two $+Q$ charges at the location of the $3Q$ charge.

ii) Taking the potential at infinity to be zero as our reference, what is the potential at the location of the $3Q$ charge?

iii) Find the kinetic energy of the $3Q$ charge as it arrives at r = infinity if it is released from rest.

i) $\vec{E}_{at\,3Q} = \vec{E}_{Top} + \vec{E}_{Bottom} = 2\left(\frac{KQ}{2a^2}\cos 45°\right)$ in x direction



ii) $V(3Q) = V_Q + V_Q$

$= \left(\frac{KQ}{\sqrt{2}a}\right)2$

iii) $\Delta KE = q\Delta V$

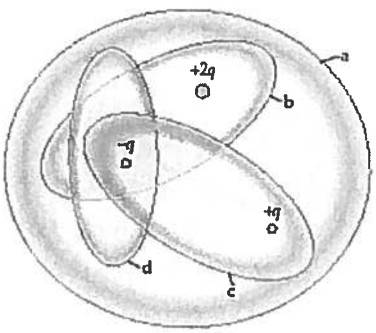
$= 3Q\left[\frac{2KQ}{\sqrt{2}a}\right]$

	P	F
2.1		
3.2		
11.1		
21.1		
23.1		
23.2		
1.3	(add)	

C. You are given three point charges, +2q, +q and –q as shown in the figure. Also in the figure are drawn four different "Gaussian" surfaces. In terms of the charges given and other known constants, find the total flux of the electric field through each of these four surfaces.

i) $\Phi_a = ?$
ii) $\Phi_b = ?$
iii) $\Phi_c = ?$
iv) $\Phi_d = ?$

Using Gauss's Law $\quad \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \phi_{tot}$



i) $\Phi_a = \frac{2q + q - q}{\epsilon_0} = \frac{2q}{\epsilon_0}$

ii) $\phi_b = \frac{2q - q}{\epsilon_0} = \frac{q}{\epsilon_0}$

iii) $\phi_c = \frac{-q + q}{\epsilon_0} = 0$

iv) $\phi_d = \frac{-q}{\epsilon_0}$

LO	P	F
3.3		
16.1		
16.2		
16.3		
16.4		

Problem I.

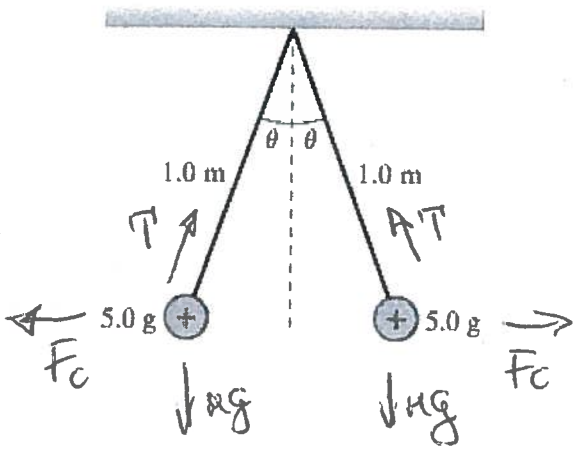
Two point masses of insulating material, each of mass 5.0 g are hung by insulating silk threads of length 1.0 m as shown in the figure below. Initially each mass is given a charge $+q$ and the silk threads supporting the insulators are observed to make an angle of θ with the vertical.

a) Draw a "free body" diagram for each of the masses shown in the figure.

b) If the angle $\theta = 15°$, find the tension in the silk threads and the Coulomb force between the charges.

c) Solve for the charges on the two insulators.

a)



b) By components

$$\sum F_x = 0 = F_c - T\sin\theta$$
$$\sum F_y = 0 = T\cos\theta - mg$$

so $T = mg/\cos\theta = .051\,N$

& $F_c = T\sin\theta = mg\tan\theta$

$= 0.0131\,N$

LO	P	F
1.1		
2.2		
3.4		
5.2		
8.1		
1.2	add	
2.3	add	

c) Solving for the charges on the two insulators.

Since $F_c = \dfrac{kq^2}{d^2} = mg\tan\theta$

Solving for $q = \sqrt{\dfrac{(mg\tan\theta)d^2}{k}}$ where $d = 2(1m\sin 15°)$

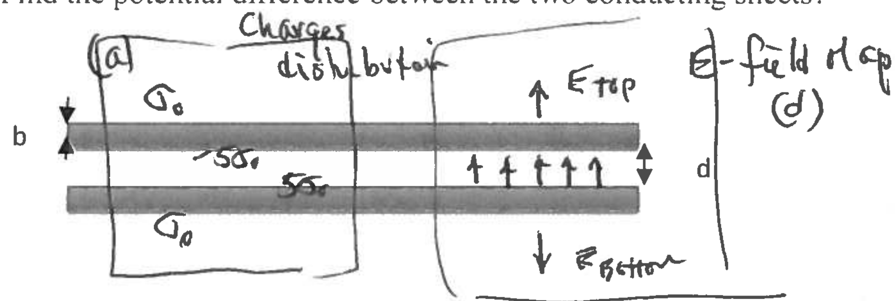
$= .517\,m$

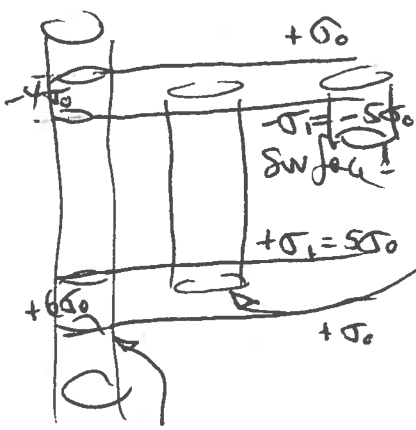
$$\boxed{q = 6.2\times10^{-7}\ \text{Coul}}$$

Problem II

You are given 2 "large" conducting sheets that have a thickness, b, in the vertical dimension. These sheets are separated (surface to surface) by a distance d as shown in the figure. The top sheet has a **net** charge density of $-4\sigma_0$ and the bottom sheet has a **net** charge density of $+6\sigma_0$. In terms of the quantities given and other known constants answer the following:

a) In the figure indicate how the charge will be distributed on the top and bottom surfaces of each conducting plate. Make sure to label the charge density on each of these surfaces.
b) Find the electric field inside each of the conducting plates.
c) Find the electric field in the regions above, between and below the conducting plates.
d) Draw the electric field lines in the three regions from part c).
e) Find the potential difference between the two conducting sheets?





a)

$\oint \vec{E}\cdot d\vec{A} = 0$ so $q_{in} = 0$

charge on bottom of top and top of bottom plates equal and opposite ($\pm\sigma_1$)

$$\oint \vec{E}\cdot dA = E_{TOP}A + E_{BOTTOM}A = \frac{2\sigma_0}{\epsilon_0}$$

$$\text{so } E_{TOP} = E_{BOTTOM} = \frac{\sigma_0}{\epsilon_0}$$

b) E-field inside a conductor is always 0!

c) $\oint \vec{E}_{\text{field TOP}}\cdot d\vec{A} = E_{TOP}A = \frac{\sigma_0 A}{\epsilon_0} \Rightarrow E_T = \frac{\sigma_0}{\epsilon_0}$ (up)

$\oint \vec{E}_{\text{field Bottom}}\cdot d\vec{A} = E_{BOTTOM}A = \frac{\sigma_0 A}{\epsilon_0} \Rightarrow E_{Bottom} = \frac{\sigma_0}{\epsilon_0}$ (down)

$\oint \vec{E}_{middle}\cdot dA = E_{middle}A = \frac{(6\sigma_0 - \sigma_0)A}{\epsilon_0}$

$E_{middle} = \frac{5\sigma_0}{\epsilon_0}$ (up)

e) $\Delta V = -\int_{Bottom}^{Top} \vec{E}\cdot d\vec{\ell} = \frac{5\sigma_0}{\epsilon_0} d$

LO	P	F
5.3		
7.2		
13.1		
13.2		
13.3		
18.1		
18.2		
18.3		
19.1		
19.2		
19.3		
19.4		
19.5		
19.6		
26.1		

Problem III.

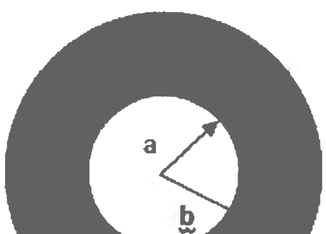
A hollow **insulating** sphere has an inner radius a and an outer radius b and contains a total charge of Q uniformly distributed throughout its volume. In terms of the quantities given, answer the following:

a) Find the electric field for the following regions:
   i. $r < a$
   ii. $a < r < b$
   iii. $r > b$

b) Taking as our reference potential, that V(r = infinity) = 0, find the potential at $V(r = b)$.

c) Keeping our reference potential at infinity, find the potential a $V(r = a)$.

d) Again keeping the reference potential at infinity the same as in the previous parts of the problem, find the potential a $V(r = 0)$.



a) Using Gauss' Law $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

i) $\vec{E}(r<a) = 0$ b/c $q_{enc} = 0$

ii) $E(a<r<b)\, 4\pi r^2 = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi (r^3-a^3) \Rightarrow E = \frac{\rho(r^3-a^3)}{3\varepsilon_0 r^2}$

with $\rho = Q/\left(\frac{4}{3}\pi(b^3-a^3)\right)$

or in terms of Q $E = \frac{kQ(r^3-a^3)}{r^2(b^3-a^3)} = \frac{kQ}{(b^3-a^3)}\left(r-\frac{a^3}{r^2}\right)$

iii) $E(>b) = \frac{kQ}{r^2}$

b) $V(r=b) - V(r=\infty) = -\int_\infty^b \vec{E}\cdot d\vec{r} = kQ\int_\infty^b \frac{1}{r^2} = \frac{kQ}{b}$

c) $V(r=a) - \underbrace{V(r=\infty)}_{0} = V(r=a) - V(r=b) + \frac{kQ}{b}$

$V(r=a) = -\int_b^a \vec{E}\cdot d\vec{r} + \frac{kQ}{b} = -\frac{kQ}{(b^3-a^3)}\int_b^a \left(r-\frac{a^3}{r^2}\right)dr + \frac{kQ}{b}$

$V(r=a) = -\frac{kQ}{(b^3-a^3)}\left[\left(\frac{a^2-b^2}{2}\right) + a^3\left(\frac{1}{a}-\frac{1}{b}\right)\right] + \frac{kQ}{b}$

In terms of ρ we get $= -\frac{\rho}{3\varepsilon_0}\int_b^a \left(r-\frac{a^3}{r^2}\right)dr + \frac{kQ}{b} = -\frac{\rho}{3\varepsilon_0}\left[\left(\frac{a^2-b^2}{2}\right) + a^3\left(\frac{1}{a}-\frac{1}{b}\right)\right] + \frac{kQ}{b}$.

d) $V(r=0) = V(r=a)$ b/c $E(r<a) = 0$!

LO	P	F
3.5		
3.6		
~~3.7~~ NOT USED		
5.4		
7.3		
7.4		
7.5		
18.4		
18.5		
18.6		
26.2		
26.3		
26.4		