Make sure to fill out the grading sheet completely including your name, exam flavor and UIN. You are allowed to write and work on this exam copy, but your answers must be bubbled in on the grading sheet to receive credit. Your bubbled responses are the only responses that will be considered for the grade.

Physics 207 Exam 1 – Flavor 1

Problem 1 (6 points) Four positive point charges (shown in the figure below) are placed at each corner of a square whose side is a. What is the net electric field produced by these four positive charges in the center of the square?

(a) $+\dfrac{2\sqrt{2}kq}{a^2}\hat{\imath}$

(b) $-\dfrac{2\sqrt{2}kq}{a^2}\hat{\imath}$

(c) $+\dfrac{2\sqrt{2}kq}{a^2}\hat{\jmath}$

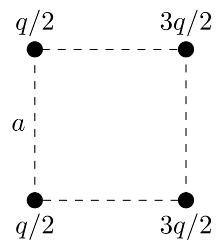
(d) $-\dfrac{2\sqrt{2}kq}{a^2}\hat{\jmath}$

(e) $+\dfrac{3\sqrt{2}kq}{a^2}\hat{\imath}$

(f) $-\dfrac{3\sqrt{2}kq}{a^2}\hat{\imath}$

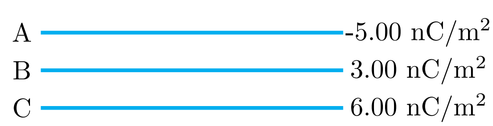
(g) $+\dfrac{3\sqrt{2}kq}{a^2}\hat{\jmath}$

(h) $-\dfrac{3\sqrt{2}kq}{a^2}\hat{\jmath}$



Problem 2 (6 points) Find the magnitude of the electric force produced by three horizontal (4.00×4.00 m^2) parallel sheets A, B, and C (from top to bottom) with the surface charge densities -5.00 nC/m^2, 3.00 nC/m^2, and 6.00 nC/m^2 accordingly on a negative charge -2.00 μC (not shown on a figure) situated 3.00 mm above the middle of sheet A. Assume that the spacing between the sheets is small compared to their size.

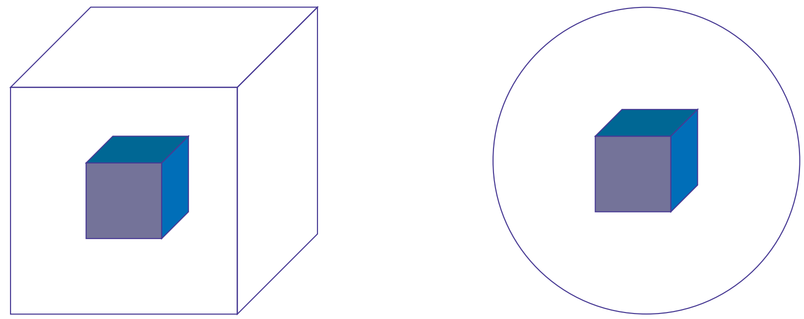
(a) 0.452 mN
(b) 0.313 mN
(c) 0.225 mN
(d) 0.184 mN
(e) 0.112 mN
(f) 0.0809 mN



Problem 3 (3 points) What is the direction of the electric force you found in the previous problem?

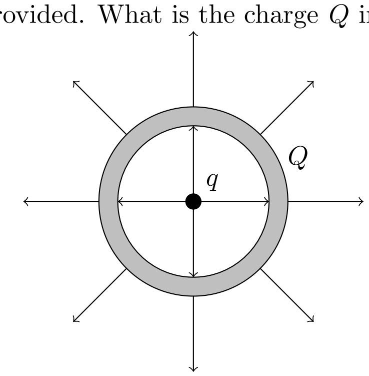
(a) Up
(b) Down
(c) Left
(d) Right
(e) 45 degrees above horizontal
(f) 45 degrees below horizontal

Problem 4 (5 points) A small dielectric cube with uniform distribution of a negative charge -q in its volume is surrounded by either a larger cube with side length d (shown on the left) or a sphere with diameter d (shown on the right). Which of the following statements is true about the flux in these two cases?



(a) The flux has a greater magnitude through the surface of the cube.
(b) The flux has a greater magnitude through the surface of the sphere.
(c) Both fluxes are equal and positive.
(d) Both fluxes are equal and negative.
(e) Both fluxes are equal and exactly zero.

Problem 5 (5 points) In the figure below, the conducting spherical shell has a total charge Q. The point charge q is at the exact center of the shell's cavity. The structure of the electric field lines is provided. What is the charge Q in terms of q?



(a) $Q = -2q$
(b) $Q = -q$
(c) $Q = -q/2$
(d) $Q = 0$
(e) $Q = +q/2$
(f) $Q = +q$
(g) $Q = +2q$

Problem 6 (10 points) A positive charge Q is distributed uniformly along a line with length a as shown in the figure. A point charge q with mass m is placed at a distance r from the right side of the line. Find the maximum speed acquired by the charge q if it was released from rest from that point. Hint: calculate the potential at $x = a + r$ (choosing a zero potential at the infinite distance), calculate the potential energy of the set of charges and use an energy conservation law.

(a) $v = \sqrt{\dfrac{2kqQ}{ma}}$

(b) $v = \sqrt{\dfrac{2kqQ}{mr}}$

(c) $v = \sqrt{\dfrac{2kqQ}{m(a+r)}}$

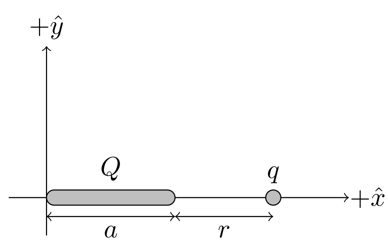
(d) $v = \sqrt{\dfrac{2kqQ}{m\left(\frac{a}{2}+r\right)}}$

(e) $v = \sqrt{\dfrac{2kqQ}{ma}\ln\left(\dfrac{r}{a}\right)}$

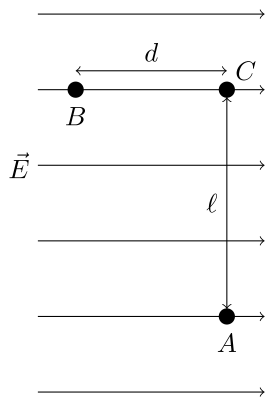
(f) $v = \sqrt{\dfrac{2kqQ}{ma}\ln\left(\dfrac{a}{r}\right)}$

(g) $v = \sqrt{\dfrac{2kqQ}{ma}\ln\left(\dfrac{r+a}{r}\right)}$

(h) $v = \sqrt{\dfrac{2kqQ}{ma}\ln\left(\dfrac{r+a}{a}\right)}$

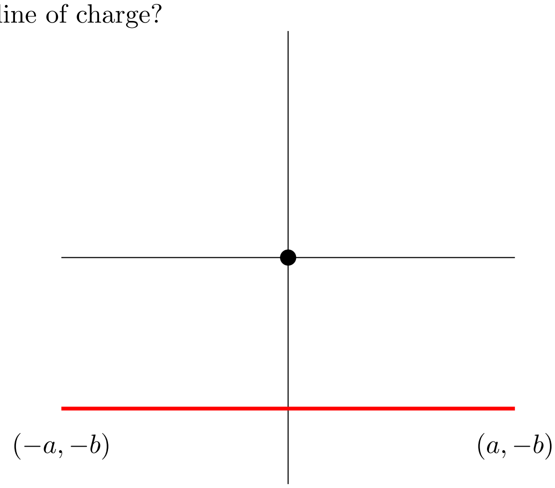


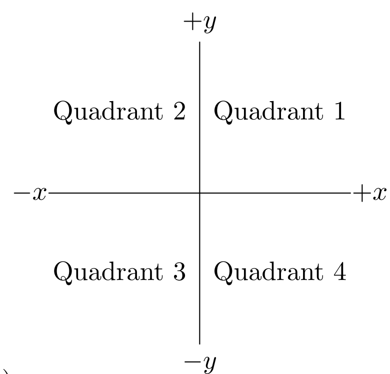
Problem 7 (8 points) There exists a uniform electric field with magnitude E. The distance between points B and C is d while the distance between points C and A is ℓ. What is the potential difference between points A and B, $\Delta V = V_A - V_B$?



(a) $+E\ell$
(b) $+Ed$
(c) $+E(\ell + d)$
(d) $+E(\ell^2 + d^2)$
(e) $-E(\ell^2 + d^2)$
(f) $-E(\ell + d)$
(g) $-Ed$
(h) $-E\ell$

Problem 8 (5 points) A non-uniform line of charge with $\lambda(x) = \alpha x$ lies in the xy-plane from $(-a, -b)$ to $(a, -b)$ as seen in the figure below. Which of the following best describes the direction of the net electric field vector at the origin due to the line of charge?





(a) $+x$
(b) Quadrant 1
(c) $+y$
(d) Quadrant 2
(e) $-x$
(f) Quadrant 3
(g) $-y$
(h) Quadrant 4

Problem 9 (8 points) A charge of $q_1 = +8.00$ nC is placed 15.0 cm from another charge $q_2 = +1.50$ nC. At what distance from q_1 along the path between the charges should a third charge, $q_3 = +4.33$ nC, be placed so that the net force on q_3 is zero?
(a) 0.023 m
(b) 0.045 m
(c) 0.056 m
(d) 0.064 m
(e) 0.086 m
(f) 0.094 m
(g) 0.105 m
(h) 0.127 m

Problem 10 (10 points) An insulating sphere has non-uniform charge distribution $\rho(r) = \alpha r^3$, total charge Q and radius R. What is the magnitude of the electric field at $r = R/2$?

(a) $\dfrac{Q}{2\pi\epsilon_0 R^2}$

(b) $\dfrac{Q}{4\pi\epsilon_0 R^2}$

(c) $\dfrac{Q}{8\pi\epsilon_0 R^2}$

(d) $\dfrac{Q}{16\pi\epsilon_0 R^2}$

(e) $\dfrac{Q}{32\pi\epsilon_0 R^2}$

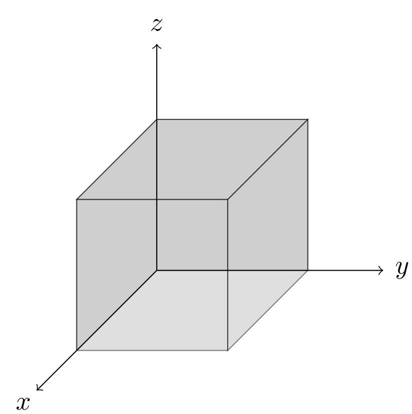
(f) $\dfrac{Q}{64\pi\epsilon_0 R^2}$

(g) $\dfrac{Q}{128\pi\epsilon_0 R^2}$

(h) $\dfrac{Q}{256\pi\epsilon_0 R^2}$

Problem 11 (8 points) A cube of side length 0.400 m is placed with one corner at the origin so that it exists in the positive x, y and z directions. There is an electric field in this region of the form $E(x, y, z) = 1.30x\hat{\imath} - 3.90z\hat{\jmath}$ in SI units. What is the net flux through the cube?

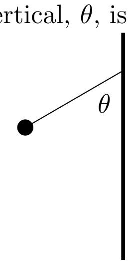
(a) +0.249 Nm^2/C
(b) +0.183 Nm^2/C
(c) +0.166 Nm^2/C
(d) +0.0832 Nm^2/C
(e) -0.0832 Nm^2/C
(f) -0.166 Nm^2/C
(g) -0.183 Nm^2/C
(h) -0.249 Nm^2/C



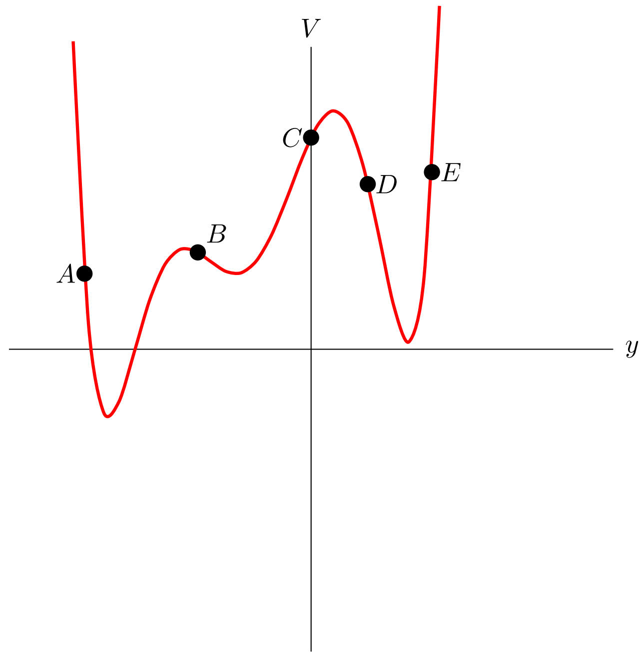
Problem 12 (8 points) A given region of space has an electric potential of the form $V(x, y) = 4x^2 - 9xy^2 + 2y^{-2}$. Which of the following describes the direction of the electric field at the point $(3, -1)$? Note that all properties are given in SI units.
(a) 47.6 degrees away from $+\hat{\jmath}$ towards $+\hat{\imath}$
(b) 75.5 degrees away from $+\hat{\imath}$ towards $+\hat{\jmath}$
(c) 3.43 degrees away from $+\hat{\jmath}$ towards $-\hat{\imath}$
(d) 61.2 degrees away from $-\hat{\imath}$ towards $+\hat{\jmath}$
(e) 14.5 degrees away from $-\hat{\jmath}$ towards $-\hat{\imath}$
(f) 42.4 degrees away from $-\hat{\imath}$ towards $-\hat{\jmath}$
(g) 28.8 degrees away from $-\hat{\jmath}$ towards $+\hat{\imath}$
(h) 86.6 degrees away from $+\hat{\imath}$ towards $-\hat{\jmath}$

Problem 13 (8 points) Two identical positive charges, both 3.0 nC, are placed: one at the origin and another at (0,4.0). How much work is done by an outside force to bring a third positive charge of magnitude 4.0 nC from an infinite distance away and place it at the location (-3.0,4.0)? All positions are given in units of meters.
(a) $+1.2 \times 10^{-8}$ J
(b) -1.2×10^{-8} J
(c) $+4.3 \times 10^{-8}$ J
(d) -4.3×10^{-8} J
(e) $+5.8 \times 10^{-8}$ J
(f) -5.8×10^{-8} J
(g) $+7.8 \times 10^{-8}$ J
(h) -7.8×10^{-8} J

Problem 14 (6 points) There exists a very large sheet with uniform charge density σ. There is an insulating string connected to the sheet and a small sphere with charge q at the end of the string. There are three forces acting on the sphere, the electric force F_E due to the sheet, the gravitational force F_g (acting downward in the figure) due to the earth and a tension force F_T due to the string. Which of the following represents the correct ranking of the magnitudes of those forces if the angle of the string relative to vertical, θ, is greater than 45 degrees?
(a) $F_E > F_g > F_T$
(b) $F_E > F_T > F_g$
(c) $F_g > F_E > F_T$
(d) $F_g > F_T > F_E$
(e) $F_T > F_E > F_g$
(f) $F_T > F_g > F_E$



Problem 15 (4 points) The figure below provides the one-dimensional electric potential as a function of y. What is the direction of the force acting on a positive charge that exists at point C?



(a) $+\hat{x}$
(b) $-\hat{x}$
(c) $+\hat{y}$
(d) $-\hat{y}$
(e) $+\hat{z}$
(f) $-\hat{z}$
(g) 0

Useful Constants:
Acceleration due to gravity: $g = 9.80$ m/s^2
Basic unit of charge: $e = 1.6 \times 10^{-19}$ C
Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
Mass of proton/neutron: $m_p = 1.67 \times 10^{-27}$ kg
Coulomb constant: $k = 8.99 \times 10^9$ Nm^2/C^2
Permittivity of free space: $\epsilon_0 = 1/(4\pi k) = 8.85 \times 10^{-12}$ C^2/Nm^2
Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ Tm/A
Speed of light in a vacuum: $c = 3 \times 10^8$ m/s
Planck's Constant: $h = 6.626 \times 10^{-34}$ Js
eV to joule conversion: 1 eV = 1.6×10^{-19} J
kilowatt-hour to joule conversion: 1 kW·hr=3.6×10^6 J
Atomic Mass Unit: 1 u = 1.66054×10^{-27} kg = 931.5 MeV/c^2

Vector Concepts:
Unit Vector: $\hat{r} = \frac{\vec{r}}{r}$
Gradient: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
Dot Product: $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$
Dot Product: $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$
Cross Product:

$$\begin{aligned} |\vec{A} \times \vec{B}| =& |\vec{A}||\vec{B}|\sin\theta \\ \vec{A} \times \vec{B} =& (A_yB_z - A_zB_y)\hat{x} - (A_xB_z - A_zB_x)\hat{y} \\ &+ (A_xB_y - A_yB_x)\hat{z} \end{aligned}$$

Sample Indefinite Integrals:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) + c$$
$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + c$$
$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}} + c$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c(n \neq -1)$$
$$\int \frac{dx}{x} = \ln(x) + c$$
$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + c$$

SI Prefixes:
T=$\times 10^{12}$, G=$\times 10^9$, M=$\times 10^6$, k=$\times 10^3$
c=$\times 10^{-2}$, m=$\times 10^{-3}$
μ=$\times 10^{-6}$, n=$\times 10^{-9}$, p=$\times 10^{-12}$, f=$\times 10^{-15}$

Useful integral relationships:
Spherical: $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$
Cylindrical (with constant r): $dV = \pi r^2 \mathrm{d}z$
Cylindrical (with constant z): $dV = z2\pi r \mathrm{d}r$
Cylindrical (with constant r): $dA = 2\pi r \mathrm{d}z$
Cylindrical (with constant z): $dA = 2\pi r \mathrm{d}r$

Geometry:
Surface Area of a Sphere: $A = 4\pi r^2$
Volume of a Sphere: $V = \frac{4}{3}\pi r^3$
Area of curved region of a cylinder: $A = 2\pi rh$
Volume of a cylinder: $V = \pi r^2 h$

Physics 1 Concepts:
Work: $W = \int \vec{F} \cdot d\vec{\ell}$
Potential Energy of conservative force:
$W_{cons} = -\Delta U$
Kinetic Energy: $K = \frac{1}{2}mv^2$
Momentum: $\vec{p} = m\vec{v}$

Chapter 21:

Coulomb's Law [N]: $\vec{F} = \dfrac{kq_1q_2}{r^2}\hat{r}$

Force due to an electric field [N]: $\vec{F} = q\vec{E}$

E Field due to a pt. charge [N/C]: $\vec{E} = \dfrac{kq}{r^2}\hat{r}$

E Field due to a continuous charge dist. [N/C]:

$\vec{E} = \displaystyle\int \frac{kdq}{r^2}\hat{r}$

Electric dipole moment [Cm]: $\vec{p} = q\vec{d}$

Torque on an electric dipole [Nm]: $\vec{\tau} = \vec{p} \times \vec{E}$

Electric pot. energy stored in electric dipole [J]:

$U = -\vec{p} \cdot \vec{E}$

Chapter 22:

Electric Flux [Vm or Nm^2/C]: $\Phi_E = \displaystyle\int \vec{E} \cdot d\vec{A}$

Electric Flux when E and θ are const.

on the surface: $\Phi_E = EA\cos\theta$

Gauss's Law (vacuum): $\Phi_E = \displaystyle\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$

Chapter 23: The below equations generally but not always assume that $V(\infty) = 0$ and/or $U(\infty) = 0$.

Elec. pot. energy between 2 pt charges [J]:

$U = \dfrac{kq_1q_2}{r}$

Elec. pot. difference btw. two locations [V or J/C]:

$\Delta V = \dfrac{\Delta U}{q}$ (or often) $V = \dfrac{U}{q}$

Electric potential due to a point charge [V]:

$V = \dfrac{kq}{r}$

Electric potential due to a charge dist. [V]:

$V = \displaystyle\int \frac{kdq}{r}$

Relating $\vec{E}$ and V: $\vec{E} = -\vec{\nabla}V$

$\Delta V = V_b - V_a = -\displaystyle\int_a^b \vec{E} \cdot d\vec{\ell}$