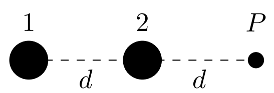
Make sure to fill out the grading sheet completely including your name, instructor, exam flavor and UIN. You are allowed to write and work on this exam copy, but your answers must be bubbled in on the grading sheet to receive credit. Your bubbled responses are the only responses that will be considered for the grade.

Physics 207 Exam 1 – Flavor 1

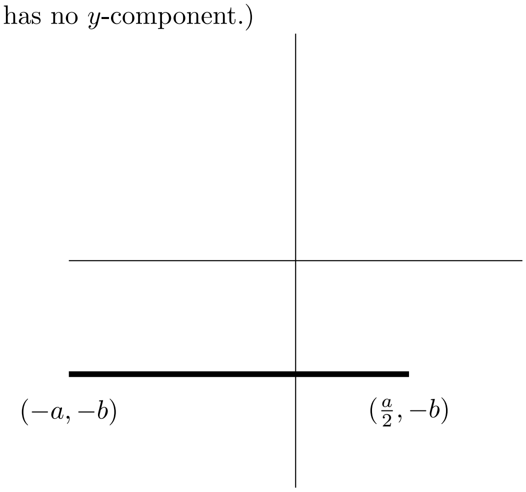
Problem 1 (6 points) There are two electrically neutral spheres. 5.40×10^{11} electrons are transferred from sphere 2 to sphere 1. As a result the net electric force on a proton at point P is 1.40×10^{-17} N. What is the distance d? Assume that the radii of the spheres are much smaller than d.

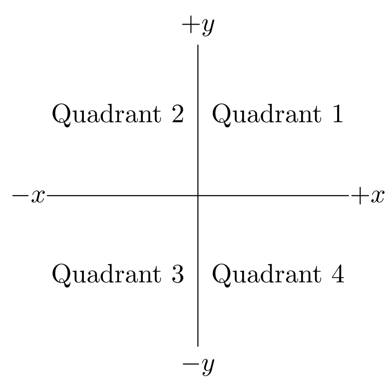


(A) 0.914 m
(B) 2.11 m
(C) 2.58 m
(D) 3.33 m
(E) 4.19 m
(F) 5.85 m
(G) 6.67 m
(H) 8.26 m

Problem 2 (2 points) What direction is the force on this proton?
(A) Right
(B) Left

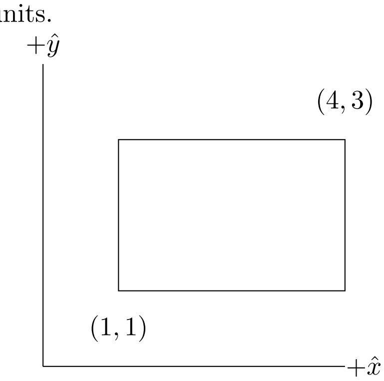
Problem 3 (4 points) A line of charge with uniform charge density $+\lambda$ exists from $(-a, -b)$ to $(a/2, -b)$. Which of the options below represents the direction of the electric field at the origin due to this line of charge? (e.g. Quadrant 1 would represent an electric field anywhere in that region, whereas $+x$ would represent an electric field ONLY pointing in $+x$ and has no y-component.)





(A) $+x$
(B) Quadrant 1
(C) $+y$
(D) Quadrant 2
(E) $-x$
(F) Quadrant 3
(G) $-y$
(H) Quadrant 4

Problem 4 (8 points) The electric field in a region of space can be described by $\vec{E} = (6x^2)\hat{\imath} + (9y^2)\hat{k}$. What is the electric flux through the rectangle in the xy-plane shown below? Note that the electric field vector and positions are given in SI units.



(A) 68 $\mathrm{Nm^2/C}$
(B) 126 $\mathrm{Nm^2/C}$
(C) 234 $\mathrm{Nm^2/C}$
(D) 432 $\mathrm{Nm^2/C}$
(E) 621 $\mathrm{Nm^2/C}$
(F) 1154 $\mathrm{Nm^2/C}$

Problem 5 (6 points) A +3 μC charged particle at location A has a kinetic energy of 240 μJ and at location B is has a kinetic energy of 150 μJ. If we know that the potential at point B is 15 V, what is the potential at point A?

(A) +105 V
(B) +75 V
(C) +45 V
(D) +15 V
(E) -15 V
(F) -45 V
(G) -75 V
(H) -105 V

Problem 6 (10 points) An insulating line of charge is along the x-axis from the origin to $x = a$ as shown in the figure. The line has a charge density $\lambda(x) = \lambda_0(x-a)^2$. Which well-defined integral below correctly represents the electric force on a charge $+q$ at the point $(-b, 0)$?

(A) $+k q \lambda_0 \int_{-b}^{a} \frac{(x-a)^2 dx}{(b-x)^2}\hat{\imath}$

(B) $+k q \lambda_0 \int_{-b}^{a} \frac{(x-a)^2 dx}{(b+x)^2}\hat{\imath}$

(C) $+k q \lambda_0 \int_{0}^{a} \frac{(x-a)^2 dx}{(b-x)^2}\hat{\imath}$

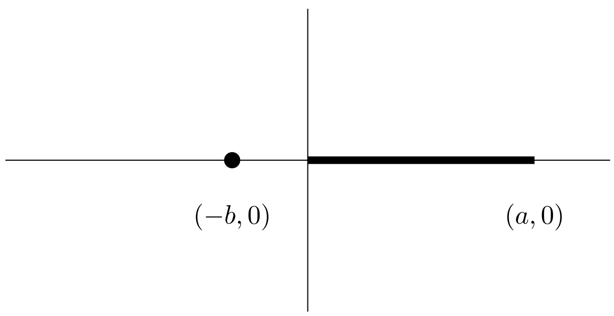
(D) $+k q \lambda_0 \int_{0}^{a} \frac{(x-a)^2 dx}{(b+x)^2}\hat{\imath}$

(E) $-k q \lambda_0 \int_{-b}^{a} \frac{(x-a)^2 dx}{(b-x)^2}\hat{\imath}$

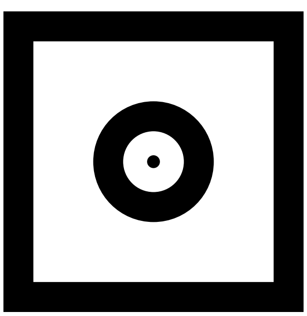
(F) $-k q \lambda_0 \int_{-b}^{a} \frac{(x-a)^2 dx}{(b+x)^2}\hat{\imath}$

(G) $-k q \lambda_0 \int_{0}^{a} \frac{(x-a)^2 dx}{(b-x)^2}\hat{\imath}$

(H) $-k q \lambda_0 \int_{0}^{a} \frac{(x-a)^2 dx}{(b+x)^2}\hat{\imath}$

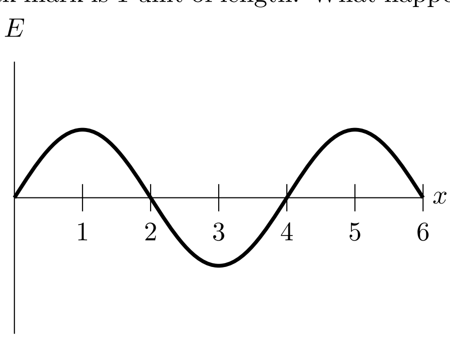


Problem 7 (4 points) In the figure below there is a point charge, -2 μC, a thick-walled spherical shell with charge -3 μC and a thick-walled cubic shell with charge 8 μC. How much charge builds up on the inside (q_{inner}) and outside (q_{outer}) surfaces of the cube?



(A) $q_{inner} = +2$ μC and $q_{outer} = -5$ μC
(B) $q_{inner} = +2$ μC and $q_{outer} = -2$ μC
(C) $q_{inner} = +2$ μC and $q_{outer} = +6$ μC
(D) $q_{inner} = +3$ μC and $q_{outer} = -2$ μC
(E) $q_{inner} = +3$ μC and $q_{outer} = +5$ μC
(F) $q_{inner} = +5$ μC and $q_{outer} = -5$ μC
(G) $q_{inner} = +5$ μC and $q_{outer} = +3$ μC

Problem 8 (4 points) An electron moves in a region of space where the electric field is sinusoidal as shown below. Each tick mark is 1 unit of length. What happens to the potential energy of this system as the electron moves from $x = 2$ to $x = 4$?

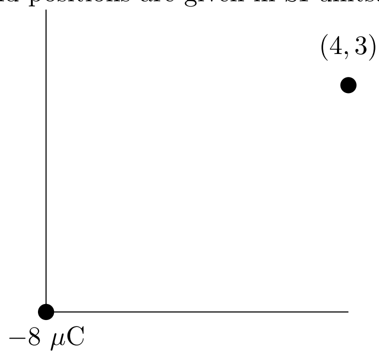


(A) The potential energy increases during the whole motion.
(B) The potential energy decreases during the whole motion.
(C) The potential energy increases and then decreases for a net change of zero.
(D) The potential energy decreases and then increases for a net change of zero.
(E) The potential energy stays constant through the whole motion.
(F) It is impossible to tell what happens to the potential energy.

Problem 9 (6 points) In a region of space, there is a function for the potential $V(x, y) = 4x^3(y - 2)$. What is the electric field vector at the position $(-2, 5)$?

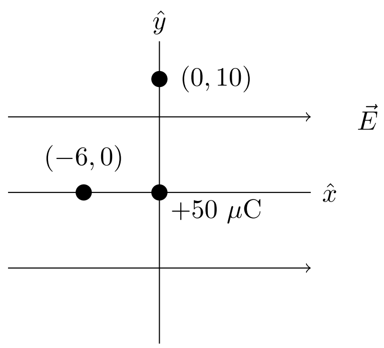
(A) $\vec{E} = +144\hat{x} - 32\hat{y}$
(B) $\vec{E} = +96\hat{x} + 96\hat{y}$
(C) $\vec{E} = +48\hat{x} + 48\hat{y}$
(D) $\vec{E} = +48\hat{x} - 80\hat{y}$
(E) $\vec{E} = -144\hat{x} + 32\hat{y}$
(F) $\vec{E} = -96\hat{x} - 96\hat{y}$
(G) $\vec{E} = -48\hat{x} - 48\hat{y}$
(H) $\vec{E} = -48\hat{x} + 80\hat{y}$

Problem 10 (10 points) A charge $-8\ \mu$C is at the origin and there is a uniform electric field $\vec{E} = 2600\hat{\jmath}$. What angle does the net electric field make at the position $(4, 3)$? Give your answer relative to the $+\hat{x}$-axis. Note that the electric field vector and positions are given in SI units.



(A) 21 degrees
(B) 37 degrees
(C) 62 degrees
(D) 159 degrees
(E) 170 degrees
(F) 194 degrees
(G) 217 degrees
(H) 339 degrees

Problem 11 (8 points) In a region of space there is an electric field $\vec{E} = 4000\hat{\imath}$. There is also a point charge $+50\ \mu$C at the origin. How much work is done by the net electric force to move another $+50\ \mu$C charge from the point $(-6, 0)$ to the point $(0, 10)$? Note that the electric field vector and positions are given in SI units.



(A) 8.2 J
(B) 5.5 J
(C) 4.5 J
(D) 3.8 J
(E) 2.7 J
(F) 1.5 J
(G) 1.2 J
(H) 0.3 J

Problem 12 (10 points) A uniformly charged insulating sphere has a radius R and a total charge $-Q$. The sphere is centered at the origin. What is the potential difference $V(R/2) - V(0)$?

(A) 0
(B) $-\dfrac{3Q}{8\pi\epsilon_0 R}$
(C) $-\dfrac{11Q}{32\pi\epsilon_0 R}$
(D) $-\dfrac{Q}{32\pi\epsilon_0 R}$
(E) ∞
(F) $+\dfrac{3Q}{8\pi\epsilon_0 R}$
(G) $+\dfrac{11Q}{32\pi\epsilon_0 R}$
(H) $+\dfrac{Q}{32\pi\epsilon_0 R}$

Problem 13 (10 points) An insulating, hollow sphere has inner radius a and outer radius b. Within the insulating material the volume charge density is given by $\rho(r) = \frac{\alpha}{r^2}$, where α is a positive constant. What is the magnitude of the electric field at a distance r from the center of the shell where $a < r < b$?

(A) $\dfrac{\alpha(b-a)}{\epsilon_0 r^2}$

(B) $\dfrac{\alpha(r-a)}{\epsilon_0 r^2}$

(C) $\left(\dfrac{\alpha}{a} - \dfrac{\alpha}{r}\right)\left(\dfrac{1}{4\pi\epsilon_0 r^2}\right)$

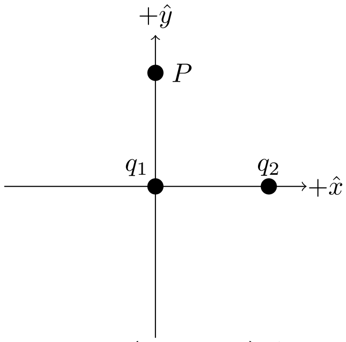
(D) $\left(\dfrac{\alpha}{a} - \dfrac{\alpha}{b}\right)\left(\dfrac{1}{4\pi\epsilon_0 r^2}\right)$

(E) $\dfrac{\alpha(b^2-a^2)}{2\epsilon_0 r^2}$

(F) $\dfrac{\alpha(r^2-a^2)}{2\epsilon_0 r^2}$

(G) 0

Problem 14 (4 points) In the figure below, you are given that the net electric field at P is in the negative x-direction and the positive y-direction. Let the distance from P to q_1 be the same as the distance from q_1 to q_2. Which single statement below MUST be true?



(A) $|q_1| = |q_2|$
(B) $|q_1| > |q_2|$
(C) $|q_1| < |q_2|$
(D) $q_1 > 0$
(E) $q_1 < 0$
(F) $q_2 > 0$
(G) $q_2 < 0$

Problem 15 (4 points) A sphere with a radius of 2 meters is centered at the origin. There are 5 protons at the position $(x, y, z) = (0, 0, 1)$, 3 electrons at the position $(x, y, z) = (1, 1, 0)$ and 7 protons at the position $(x, y, z) = (3, 2, 1)$. What is the flux through this sphere?

(A) $+\dfrac{15e}{\epsilon_0}$

(B) $+\dfrac{9e}{\epsilon_0}$

(C) $+\dfrac{8e}{\epsilon_0}$

(D) $+\dfrac{5e}{\epsilon_0}$

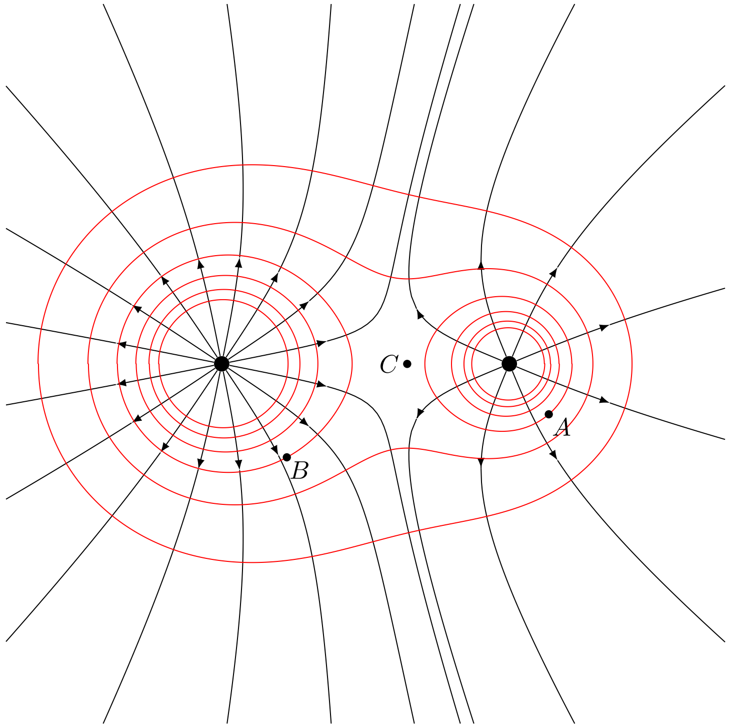
(E) $+\dfrac{2e}{\epsilon_0}$

(F) $-\dfrac{2e}{\epsilon_0}$

(G) $-\dfrac{3e}{\epsilon_0}$

(H) $-\dfrac{4e}{\epsilon_0}$

Problem 16 (4 points) In the figure below, there are three labeled points. Which point or points have the largest potential assuming that $V = 0$ infinitely far away?



(A) Point A has the largest potential.
(B) Point B has the largest potential.
(C) Point C has the largest potential.
(D) Points A and B are identical and have the largest potential.
(E) Points A and C are identical and have the largest potential.
(F) Points B and C are identical and have the largest potential.
(G) Points A, B and C are all identical and as such have the largest potential.

Scratch Paper

Useful Constants:
Acceleration due to gravity: $g = 9.80$ m/s^2
Basic unit of charge: $e = 1.6 \times 10^{-19}$ C
Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
Mass of proton/neutron: $m_p = 1.67 \times 10^{-27}$ kg
Coulomb constant: $k = 8.99 \times 10^9$ Nm^2/C^2
Permittivity of free space: $\epsilon_0 = 1/(4\pi k) = 8.85 \times 10^{-12}$ C^2/Nm^2
Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ Tm/A
Speed of light in a vacuum: $c = 3 \times 10^8$ m/s
Planck's Constant: $h = 6.626 \times 10^{-34}$ Js
eV to joule conversion: 1 eV = 1.6×10^{-19} J
kilowatt-hour to joule conversion: 1 kW·hr=3.6×10^6 J
Atomic Mass Unit: 1 u = 1.66054×10^{-27} kg = 931.5 MeV/c^2

Vector Concepts:
Unit Vector: $\hat{r} = \frac{\vec{r}}{r}$
Gradient: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
Dot Product: $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$
Dot Product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
Cross Product:

$$\begin{aligned}|\vec{A} \times \vec{B}| =& |\vec{A}||\vec{B}|\sin\theta \\ \vec{A} \times \vec{B} =& (A_y B_z - A_z B_y)\hat{x} - (A_x B_z - A_z B_x)\hat{y} \\ &+ (A_x B_y - A_y B_x)\hat{z}\end{aligned}$$

Sample Indefinite Integrals:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) + c$$
$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + c$$
$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}} + c$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c(n \neq -1)$$
$$\int \frac{dx}{x} = \ln(x) + c$$
$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{1}{x} + c$$
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + c$$

SI Prefixes:
T=$\times 10^{12}$, G=$\times 10^9$, M=$\times 10^6$, k=$\times 10^3$
c=$\times 10^{-2}$, m=$\times 10^{-3}$
μ=$\times 10^{-6}$, n=$\times 10^{-9}$, p=$\times 10^{-12}$, f=$\times 10^{-15}$

Useful integral relationships:
Spherical: $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$
Cylindrical (with constant r): $dV = \pi r^2 \mathrm{d}z$
Cylindrical (with constant z): $dV = z 2\pi r \mathrm{d}r$
Cylindrical (with constant r): $dA = 2\pi r \mathrm{d}z$
Cylindrical (with constant z): $dA = 2\pi r \mathrm{d}r$

Geometry:
Surface Area of a Sphere: $A = 4\pi r^2$
Volume of a Sphere: $V = \frac{4}{3}\pi r^3$
Area of curved region of a cylinder: $A = 2\pi r h$
Volume of a cylinder: $V = \pi r^2 h$

Physics 1 Concepts:
Work: $W = \int \vec{F} \cdot d\vec{\ell}$
Potential Energy of conservative force:
$W_{cons} = -\Delta U$
Kinetic Energy: $K = \frac{1}{2}mv^2$
Momentum: $\vec{p} = m\vec{v}$

Chapter 21:

Coulomb's Law [N]: $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$

Force due to an electric field [N]: $\vec{F} = q\vec{E}$

E Field due to a pt. charge [N/C]: $\vec{E} = \frac{kq}{r^2}\hat{r}$

E Field due to a continuous charge dist. [N/C]:
$\vec{E} = \int \frac{kdq}{r^2}\hat{r}$

Electric dipole moment [Cm]: $\vec{p} = q\vec{d}$

Torque on an electric dipole [Nm]: $\vec{\tau} = \vec{p} \times \vec{E}$

Electric pot. energy stored in electric dipole [J]:
$U = -\vec{p} \cdot \vec{E}$

Chapter 22:

Electric Flux [Vm or Nm^2/C]: $\Phi_E = \int \vec{E} \cdot d\vec{A}$

Electric Flux when E and θ are const.
on the surface: $\Phi_E = EA\cos\theta$

Gauss's Law (vacuum): $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$

Chapter 23: The below equations generally but not always assume that $V(\infty) = 0$ and/or $U(\infty) = 0$.

Elec. pot. energy between 2 pt charges [J]:
$U = \frac{kq_1q_2}{r}$

Elec. pot. difference btw. two locations [V or J/C]:
$\Delta V = \frac{\Delta U}{q}$ (or often) $V = \frac{U}{q}$

Electric potential due to a point charge [V]:
$V = \frac{kq}{r}$

Electric potential due to a charge dist. [V]:
$V = \int \frac{kdq}{r}$

Relating $\vec{E}$ and V: $\vec{E} = -\vec{\nabla}V$

$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$