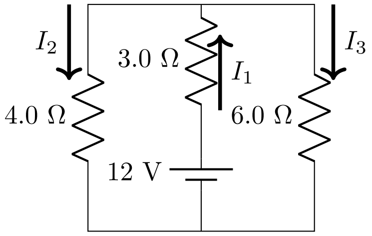
Make sure to fill out the grading sheet completely including your name, instructor, exam flavor and UIN. You are allowed to write, work on and keep this exam copy, but your answers must be bubbled in on the grading sheet to receive credit.

Physics 207 Comprehensive Exam – Flavor 1

Problem 1: (6 Points) For the circuit below, determine the current labeled I_2.



(A) 0.89
(B) 1.3
(C) 2.2
(D) 1.8
(E) 2.8

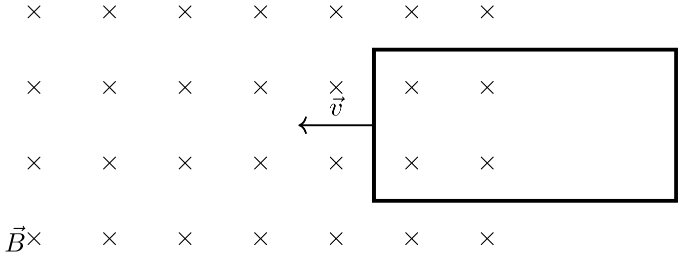
Problem 2: (6 Points) Three charges of +5.00 pC, +2.00 pC and -3.00 pC are at the corners of an equilateral triangle with side length 1.75 m. What is the total potential energy stored in this system of three charges?

(A) $+2.79 \times 10^{-13}$ J
(B) $+1.59 \times 10^{-13}$ J
(C) $+8.52 \times 10^{-14}$ J
(D) -3.23×10^{-14} J
(E) -5.66×10^{-14} J

Problem 3: (6 points) What is the magnitude of the net force that acts on a particle with charge $q = 0.5$ C at the instant when it has a velocity vector $\vec{v} = 1000\hat{x}$ at a location in space with a magnetic field $\vec{B} = 0.250\hat{x} - 0.250\hat{y}$ and an electric field $\vec{E} = +450\hat{y} - 800\hat{z}$? All vectors are given in SI units.

(A) 125 N
(B) 245 N
(C) 459 N
(D) 571 N
(E) 773 N

Problem 4: (3 Points) In the figure below the loop is moving with a velocity $\vec{v}$ to the left. The magnetic field is into the page. What is the direction of the induced current in the loop when it is still only partially in the field as drawn below?

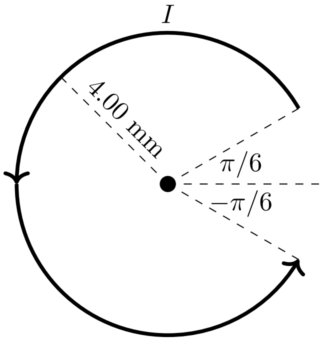


(A) Clockwise
(B) Counterclockwise
(C) The induced current will be zero.

Problem 5: (6 points) An RL-circuit with $L = 3.00$ H and an RC-circuit with $C = 3.00$ mF have the same time constant. How long will it take to charge the capacitor in the RC-circuit to 60% of its maximum value?

(A) 0.35×10^{-2} s
(B) 2.90×10^{-2} s
(C) 4.88×10^{-2} s
(D) 6.59×10^{-2} s
(E) 8.69×10^{-2} s

Problem 6: (6 points) A partial ring with radius 4.00 mm is carrying 150 mA of current in a counterclockwise direction as shown in the figure below. What is the magnetic field at the center of the ring?

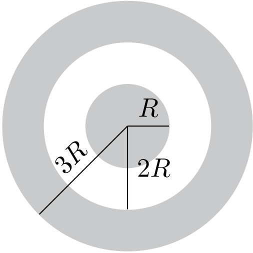


(A) 6.53×10^{-6} T
(B) 3.93×10^{-6} T
(C) 1.96×10^{-5} T
(D) 2.19×10^{-4} T
(E) 9.82×10^{-4} T
(F) 4.91×10^{-3} T

Problem 7: (2 Points) In the previous problem, what is the direction of the magnetic field created at the center of the ring?

(A) Up
(B) Down
(C) Right
(D) Left
(E) Out of the page
(F) Into the page

Problem 8: (4 Points) A solid insulating sphere of radius R is uniformly charged with a total charge $+6Q$. This is surrounded by a concentric, conducting spherical shell that has total charge $-2Q$. This shell has inner radius $2R$ and outer radius $3R$. How much charge builds up on the outer surface of the conducting shell?

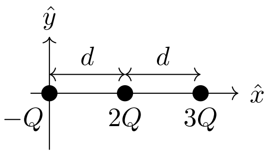


(A) $+8Q$
(B) $+4Q$
(C) $+2Q$
(D) $-2Q$
(E) $-4Q$
(F) $-8Q$

Problem 9: (4 Points) A parallel-plate capacitor is connected to the battery. You slide a slab of dielectric with a dielectric constant κ completely filling the space between the plates of a parallel-plate capacitor. What effect does adding the dielectric have on the amount of charge Q on the plates and energy U stored in the capacitor? Answer this problem assuming the battery stayed connected to the capacitor the entire time.

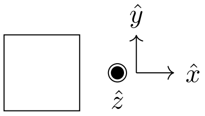
(A) Q increases by a factor κ and U decreases by a factor κ
(B) Q increases by a factor κ and U increases by a factor κ
(C) Q decreases by a factor κ and U decreases by a factor κ
(D) Q decreases by a factor κ and U increases by a factor κ
(E) Q increases by a factor κ and U decreases by a factor κ^2
(F) Q increases by a factor κ and U increases by a factor κ^2
(G) Q decreases by a factor κ and U decreases by a factor κ^2
(H) Q decreases by a factor κ and U increases by a factor κ^2

Problem 10: (6 points) A system of three point charges, $-Q$, $2Q$ and $3Q$, are arranged on the x-axis as shown. The spacing between the charges is d. In terms of the quantities given, what is the total electric force (the magnitude and direction) acting on the $3Q$ charge?



(A) $\frac{5}{2}\frac{kQ^2}{d^2}$ left
(B) $\frac{5}{2}\frac{kQ^2}{d^2}$ right
(C) $\frac{9}{2}\frac{kQ^2}{d^2}$ left
(D) $\frac{9}{2}\frac{kQ^2}{d^2}$ right
(E) $\frac{21}{4}\frac{kQ^2}{d^2}$ left
(F) $\frac{21}{4}\frac{kQ^2}{d^2}$ right
(G) $\frac{27}{4}\frac{kQ^2}{d^2}$ left
(H) $\frac{27}{4}\frac{kQ^2}{d^2}$ right

Problem 11: (6 points) A square loop with sides of 5.00 cm is in the xy-plane as shown below. At $t = 0$ there is a magnetic field of $\vec{B}_0 = 0.030\hat{z}$. During a span of 15 seconds, the magnetic field changes to $\vec{B} = -0.180\hat{z}$. What is the magnitude of the average induced emf in the loop during this time frame?



(A) 6.50×10^{-5} V
(B) 5.50×10^{-5} V
(C) 4.50×10^{-5} V
(D) 3.50×10^{-5} V
(E) 2.50×10^{-5} V

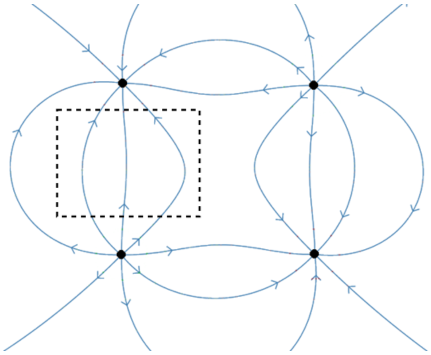
Problem 12: (6 Points) The intensity of light from the sun that hits a satellite that is orbiting earth is 1.4 kW/m^2. The area of the satellite's solar panels is 4 m^2 and is oriented perpendicular to an incident radiation. What is the radiation power received by a satellite and how long does it take to acquire 56 kJ of a radiation energy?

(A) $P = 5600$ W and $t = 10$ s
(B) $P = 0.35$ W and $t = 625$ s
(C) $P = 18$ W and $t = 45$ s
(D) $P = 5600$ W and $t = 1$ s
(E) $P = 0.35$ W and $t = 6.25$ s
(F) $P = 18$ W and $t = 3000$ s

Problem 13: (6 Points) The potential in a region of space can be described by the function: $V(x) = 12x^2 + 23x - 77$. What is the electric field vector $\vec{E}$ at position $x = -3$?

(A) $+49\hat{x}$
(B) $+38\hat{x}$
(C) 0
(D) $-38\hat{x}$
(E) $-49\hat{x}$

The following prompt and figure will be used for Problems 14 and 15. The four points all represent point charges of equal magnitude.



Problem 14: (3 Points) What is the sign of the charge in the bottom right corner of the figure below?

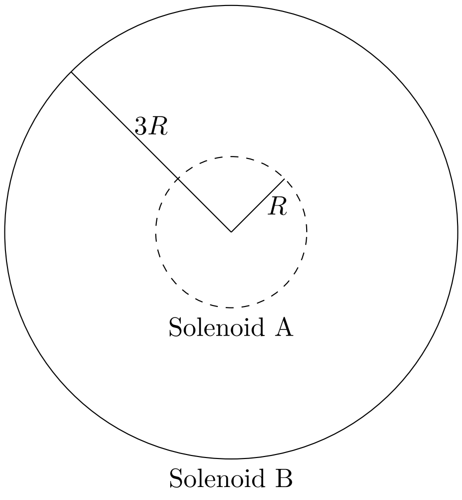
(A) Positive
(B) Negative
(C) Zero
(D) It is impossible to tell

Problem 15: (3 Points) What is the sign of the flux through the Gaussian surface represented by the dashed line?

(A) Positive
(B) Negative
(C) Zero
(D) It is impossible to tell

Problem 16: (6 points) Solenoid A has a circular cross section with radius R, length ℓ and N turns. This solenoid is directly inside solenoid B which has a circular cross section with radius $3R$, length ℓ and number of turns $5N$. What is the mutual inductance of this system? Treat the length ℓ as large compared to the radii and you should ignore any edge effects in this system.

Top view of the nested solenoid system



(A) $3\frac{\mu_0 \pi N^2 R^2}{\ell}$

(B) $5\frac{\mu_0 \pi N^2 R^2}{\ell}$

(C) $9\frac{\mu_0 \pi N^2 R^2}{\ell}$

(D) $15\frac{\mu_0 \pi N^2 R^2}{\ell}$

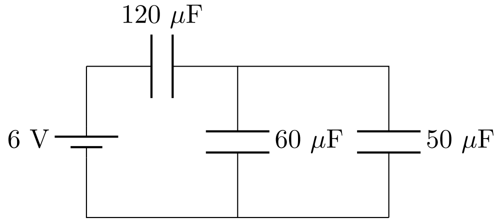
(E) $25\frac{\mu_0 \pi N^2 R^2}{\ell}$

(F) $45\frac{\mu_0 \pi N^2 R^2}{\ell}$

(G) $75\frac{\mu_0 \pi N^2 R^2}{\ell}$

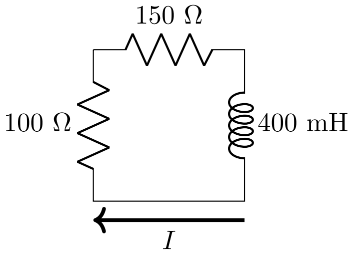
(H) $225\frac{\mu_0 \pi N^2 R^2}{\ell}$

Problem 17: (6 Points) What is the equivalent capacitance of the circuit below?



(A) 147 μF
(B) 92.8 μF
(C) 73.6 μF
(D) 57.4 μF
(E) 17.4 μF

Problem 18: (6 Points) At one instant in time the current in the circuit below is 20.0 mA. What is the rate at which the current is changing at that instant?

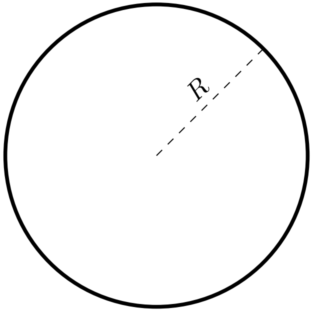


(A) 25.0 A/s
(B) 12.5 A/s
(C) 0.0 A/s
(D) -12.5 A/s
(E) -25.0 A/s

Problem 19: (3 Points) An electromagnetic wave has an electric field in $+\hat{x}$ an a magnetic field in $+\hat{z}$, what direction is the wave traveling?

(A) $+\hat{x}$
(B) $-\hat{x}$
(C) $+\hat{y}$
(D) $-\hat{y}$
(E) $+\hat{z}$
(F) $-\hat{z}$

Problem 20: (6 Points) A uniformly charged ring has a radius R and total charge Q. What is the magnitude of the electric field and what is the potential at the center of the ring?



(A) $E = \dfrac{kQ}{R^2}$ and $V = \dfrac{kQ}{R}$

(B) $E = \dfrac{kQ}{R^2}$ and $V = 0$

(C) $E = 0$ and $V = \dfrac{kQ}{R}$

(D) $E = 0$ and $V = 0$

Useful Constants:

Acceleration due to gravity: $g = 9.80$ m/s^2

Basic unit of charge: $e = 1.6 \times 10^{-19}$ C

Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg

Mass of proton/neutron: $m_p = 1.67 \times 10^{-27}$ kg

Coulomb constant: $k = 8.99 \times 10^9$ Nm^2/C^2

Permittivity of free space: $\epsilon_0 = 1/(4\pi k) = 8.85 \times 10^{-12}$ C^2/Nm^2

Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ Tm/A

Speed of light in a vacuum: $c = 3 \times 10^8$ m/s

Planck's Constant: $h = 6.626 \times 10^{-34}$ Js

eV to joule conversion: 1 eV = 1.6×10^{-19} J

kilowatt-hour to joule conversion: 1 kW·hr=3.6×10^6 J

Atomic Mass Unit: 1 u = 1.66054×10^{-27} kg = 931.5 MeV/c^2

Vector Concepts:

Unit Vector: $\hat{r} = \frac{\vec{r}}{r}$

Gradient: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$

Dot Product: $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

Dot Product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Cross Product:

$$\begin{aligned} |\vec{A} \times \vec{B}| =& |\vec{A}||\vec{B}| \sin\theta \\ \vec{A} \times \vec{B} =& (A_y B_z - A_z B_y)\hat{x} - (A_x B_z - A_z B_x)\hat{y} \\ &+ (A_x B_y - A_y B_x)\hat{z} \end{aligned}$$

Sample Indefinite Integrals:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) + c$$

$$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + c$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}} + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c (n \neq -1)$$

$$\int \frac{dx}{x} = \ln(x) + c$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{1}{x} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c$$

SI Prefixes:

T=$\times 10^{12}$, G=$\times 10^9$, M=$\times 10^6$, k=$\times 10^3$

c=$\times 10^{-2}$, m=$\times 10^{-3}$

μ=$\times 10^{-6}$, n=$\times 10^{-9}$, p=$\times 10^{-12}$, f=$\times 10^{-15}$

Useful integral relationships:

Spherical: $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$

Cylindrical (constant over r): $dV = \pi r^2 \mathrm{d}z$

Cylindrical (constant over z): $dV = z 2\pi r \mathrm{d}r$

Cylindrical (with constant r): $dA = 2\pi r \mathrm{d}z$

Cylindrical (with constant z): $dA = 2\pi r \mathrm{d}r$

Geometry:

Area of a Sphere: $A = 4\pi r^2$

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Area of curved region of a cylinder: $A = 2\pi r h$

Volume of a cylinder: $V = \pi r^2 h$

Physics 1 Concepts:

Work: $W = \int \vec{F} \cdot d\vec{\ell}$

Kinetic Energy: $K = \frac{1}{2}mv^2$

Momentum: $\vec{p} = m\vec{v}$

Chapter 21:

Coulomb's Law [N]: $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$

Force due an electric field [N]: $\vec{F} = q_0\vec{E}$

E Field Due to a pt. charge [N/C]: $\vec{E} = \frac{kq}{r^2}\hat{r}$

E Field Due to a cont. charge dist. [N/C]:

$\vec{E} = \int \frac{kdq}{r^2}\hat{r}$

Electric dipole moment [Cm]: $\vec{p} = q\vec{d}$

Torque on an electric dipole [Nm]: $\vec{\tau} = \vec{p} \times \vec{E}$

Electric pot. ene. stored in electric dipole [J]:

$U = -\vec{p} \cdot \vec{E}$

Chapter 22:

Electric Flux [Vm or Nm^2/C]: $\Phi_E = \int \vec{E} \cdot d\vec{A}$

Electric Flux when E and θ are const.

on the surface: $\Phi_E = EA\cos\theta$

Gauss's Law (vacuum): $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$

Chapter 23: The below equations generally but not always assume that $V(\infty) = 0$ and/or $U(\infty) = 0$.

Elec. pot. energy between 2 pt charges [J]:

$U = \frac{kqq_0}{r}$

Elec. pot. difference btw. two locations [V or J/C]:

$\Delta V = \frac{\Delta U}{q_0}$ (or often) $V = \frac{U}{q_0}$

Electric potential due to a point charge [V]:

$V = \frac{kq}{r}$

Electric potential Due to a charge dist [V]:

$V = \int \frac{kdq}{r}$

Relating $\vec{E}$ and V: $\vec{E} = -\vec{\nabla}V$

$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{\ell}$

Chapter 24:

Capacitance [F]: $Q = CV$

Capacitance for Parallel Plates [F]: $C = \frac{\kappa\epsilon_0 A}{d}$

Energy stored in a capacitor [J]: $U = \frac{1}{2}CV^2$

E field energy density [J/m^3]: $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$

Definition of Dielectric Constant:

$E = \frac{E_0}{\kappa}$, $V = \frac{V_0}{\kappa}$, $C = \kappa C_0$

Eff. Cap. (series) [F]: $\frac{1}{C_{eff}} = \sum_i \frac{1}{C_i}$

Eff. Cap. (parallel) [F]: $C_{eff} = \sum_i C_i$

Chapters 25 and 26:

Electric Current [A]: $I = \frac{dq}{dt}$

I from current density [A]: $I = \int \vec{j} \cdot d\vec{A}$

j of uniform current [A/m^2]: $|\vec{j}| = \frac{I}{A}$

j for charges in motion [A/m^2]: $\vec{j} = nq\vec{v}_d$

Ohm's Law: $\vec{E} = \rho\vec{j}$

Ohm's Law: $\Delta V = IR$ (or often just) $V = IR$

Resistivity and conductivity: $\rho = \frac{1}{\sigma}$

Resistance of a wire [Ω]: $R = \frac{\rho\ell}{A}$

Resistance of an object [Ω]: $R = \int \frac{\rho(x)dx}{A(x)}$

Power in a circuit element [W]:

$P = I\Delta V$ (or often) $P = IV$

Eff. Res. (series) [Ω]: $R_{eff} = \sum_i R_i$

Eff. Res. (parallel) [Ω]: $\frac{1}{R_{eff}} = \sum_i \frac{1}{R_i}$

Time constant for an RC-circuit [s]: $\tau = RC$

Charge on a charging capacitor [C]:

$q(t) = q_{\max}(1 - e^{-t/\tau})$

Charge on a discharging capacitor [C]:

$q(t) = q_0 e^{-t/\tau}$

Current in an RC-circuit [A]: $I(t) = I_0 e^{-t/\tau}$

Chapter 27:

Mag. Force on a moving q [N]: $\vec{F} = q\vec{v} \times \vec{B}$

Mag. Force on a current-carrying conductor [N]:

$\vec{F} = I \int d\vec{\ell} \times \vec{B}$

R of q's path in a B field [m]: $R = \dfrac{mv}{|q|B}$

Magnetic Dipole Moment [Am^2]: $\vec{\mu} = I\vec{A}$

Torque on current loops [Nm]: $\vec{\tau} = N\vec{\mu} \times \vec{B}$

Mag. pot. ene. in a magnetic dipole [J]:

$U = -N\vec{\mu} \cdot \vec{B}$

Chapter 28:

Biot-Savart Law (2 forms):

B made by a moving charge [T]: $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\vec{v} \times \hat{r}}{r^2}$

B made by any current [T]: $\vec{B} = \int \dfrac{\mu_0}{4\pi} \dfrac{Id\vec{\ell} \times \hat{r}}{r^2}$

B made by a long straight wire in a vacuum [T]:

$B = \dfrac{\mu_0 I}{2\pi r}$

B made by N loops, w/ radius R, on the axis,

z from the center (vacuum) [T]: $B = \dfrac{N\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$

B made inside a solenoid: $B = \mu_0 K_m \dfrac{N}{\ell} I$

Chapter 29:

Magnetic Flux [Wb]: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Magnetic Flux when B and θ are const.

on the surface: $\Phi_B = BA\cos\theta$

Faraday's Law [V]: $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$

Motional emf [V]: $\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$

Induced E Fields: $\oint \vec{E} \cdot d\vec{\ell} = -\dfrac{d\Phi_B}{dt}$

Displacement current [A]: $i_d = \epsilon_0 \dfrac{d\Phi_E}{dt}$

General Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I_{\text{encl}} + i_d)$

Chapter 30:

emf and Mutual Inductance:

$\mathcal{E}_1 = -M\dfrac{di_2}{dt}$, $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$

Mut. Inductance [H]: $M = \dfrac{N_2\Phi_{B,2}}{i_1} = \dfrac{N_1\Phi_{B,1}}{i_2}$

emf and Self Inductance: $\mathcal{E} = -L\dfrac{di}{dt}$

Self Inductance [H]: $L = \dfrac{N\Phi_B}{i}$

Inductance of a Solenoid [H]: $L = \dfrac{\mu_0 N^2 A}{\ell}$

Magnetic energy stored in an inductor [J]:

$U = \dfrac{1}{2}LI^2$

B field energy density [J/m^3]: $u_B = \dfrac{1}{2\mu_0}B^2$

Time Constant in an RL-Circuit [s]: $\tau = L/R$

Current growth in an RL-Circuit [A]:

$i(t) = I_{max}(1 - e^{-t/\tau})$

Current decay in an RL-Circuit [A]:

$i(t) = I_0 e^{-t/\tau}$

Angular frequency of the oscillation

in an LC-Circuit [rad/s]: $\omega = \dfrac{1}{\sqrt{LC}}$

q on a capacitor in an ideal LC-Circuit [C]:

$q(t) = q_{max}\cos(\omega t + \phi)$

Chapter 32:

Wave speed [m/s]: $v = f\lambda$

Wave number [1/m]: $k = \dfrac{2\pi}{\lambda}$

E-B relationship: $E = cB$

Speed of light (vacuum): $c = \dfrac{1}{\sqrt{\epsilon_0\mu_0}}$

Speed of light (medium): $v = \dfrac{c}{\sqrt{\kappa K_m}}$

Poynting Vector [W/m^2]: $\vec{S} = \dfrac{1}{\mu_0}\vec{E} \times \vec{B}$

Intensity of a wave in vacuum [W/m^2]:

$I = uc = \dfrac{P}{A}$

Total EM energy density [J/m^3]: $u = u_E + u_B$

Scratch Paper

Scratch Paper