Make sure to fill out the grading sheet completely including your name, instructor, exam flavor and UIN. You are allowed to write, work on and keep this exam copy, but your answers must be bubbled in on the grading sheet to receive credit.

# Physics 207 Exam 2 – Flavor 1

**Problem 1: (3 points)** A student wants to solve for the current in each branch of the circuit below. How many independent equations are needed?

- (A) 3 equations(B) 4 equations(C) 5 equations(D) 6 equations
- (E) 7 equations
- (F) 8 equations
- (G) 9 equations
- $(\mathbf{U})$  If  $\mathbf{i}_{\mathbf{U}}$  is a satisfier of  $\mathbf{U}$
- (H) It is impossible to tell



**Problem 2:** (8 points) Define  $\Delta V = V_B - V_A$ . What is  $\Delta V$  given points A and B in the circuit below?



**Problem 3:** (8 points) In a discharging *RC*-Circuit, it takes 12.5 seconds for the current through the circuit to reach 70% of the maximum possible current. If the resistor in the circuit is 1500  $\Omega$ , what is the capacitor?

(A)  $6.92 \times 10^{-3}$  F (B)  $2.34 \times 10^{-2}$  F (C)  $5.24 \times 10^{-2}$  F (D) 0.591 F (E) 1.85 F (F) 10.4 F (G) 24.9 F (H) 35.0 F The capacitor network below will be used for problems 4 and 5. All capacitors in the circuit have a capacitance of C (subscripts are for identification purposes only).



Problem 4: (8 points) What is the effective capacitance of this circuit?



**Problem 5:** (8 points) Assume that the total charge stored in the network is Q. What are the charges on  $C_1$  and  $C_4$  in terms of Q?

Option	$C_1$	$C_4$
(A)	$\frac{1}{3}Q$	$\frac{1}{2}Q$
(B)	$\frac{1}{3}Q$	Q
(C)	$\frac{1}{2}Q$	$\frac{1}{2}Q$
(D)	$\frac{2}{3}Q$	$\frac{1}{2}Q$
(E)	$\frac{2}{3}Q$	Q
(F)	Q	$\frac{1}{3}Q$
(G)	Q	$\frac{2}{3}Q$
(H)	Q	Q

**Problem 6:** (8 points) A capacitor in a circuit uses square plates with side lengths of 4.00 mm. The plates are separated by a distance of 0.600 mm. When there is a potential difference of 6.00 V applied across the plates,  $5.00 \times 10^{-11}$  J of energy is stored in the capacitor. What is the dielectric constant of the material in the capacitor?

(A) 11.8

(B) 6.96

(C) 5.89

(D) 3.48

(E) 2.94

(F) 1.47

The following description will be used for Problems 7 and 8. A 12 V battery with an internal resistance of 0.5  $\Omega$  is connected to two resistors which are in parallel with each other. The parallel resistors are 2 and 3  $\Omega$  respectively.

**Problem 7: (8 points)** What is the terminal voltage that the battery supplies to the circuit?

 $\begin{array}{l} (A) \ V = 15.5 \ V \\ (B) \ V = 12.0 \ V \\ (C) \ V = 10.9 \ V \\ (D) \ V = 9.88 \ V \\ (E) \ V = 8.47 \ V \\ (F) \ V = 7.93 \ V \\ (G) \ V = 5.35 \ V \\ (H) \ V = 3.45 \ V \end{array}$ 

**Problem 8:** (8 points) How much power is dissipated in the 3  $\Omega$  resistor?

(A) 134 W
(B) 84.7 W
(C) 78.6 W
(D) 42.2 W
(E) 39.7 W
(F) 35.9 W
(G) 32.6 W
(H) 23.9 W

**Problem 9:** (8 points) In the circuit below, the capacitor is uncharged before the switch is closed at t = 0. After a very long time, what is the charge on the capacitor?



**Problem 10:** (8 points) A cylindrical resistor is connected to a voltage source  $V_0$ . The cylinder has length  $\ell$  and radius b. The material has a variable resistivity  $\rho(x) = \rho_0 \frac{x}{\ell}$ . Find the current through the resistor.



$$\begin{aligned} \text{(A)} \ I &= \frac{\pi b^2 V_0}{2\rho_0} \\ \text{(B)} \ I &= \frac{\pi b^2 V_0}{\rho_0} \\ \text{(C)} \ I &= \frac{2\pi b^2 V_0}{\rho_0} \\ \text{(D)} \ I &= \frac{3\pi b^2 V_0}{\rho_0} \\ \text{(E)} \ I &= \frac{\pi b^2 V_0}{2\rho_0 \ell} \\ \text{(F)} \ I &= \frac{\pi b^2 V_0}{\rho_0 \ell} \\ \text{(G)} \ I &= \frac{2\pi b^2 V_0}{\rho_0 \ell} \\ \text{(H)} \ I &= \frac{3\pi b^2 V_0}{\rho_0 \ell} \end{aligned}$$

**Problem 11:** (7 points) In the circuit below, all capacitors are uncharged before the switch is closed at t = 0. Let the capacitances  $C_1 = 2C$  and  $C_2 = C$ . What is the charge on the two capacitors a long time after the switch is closed  $(t \to \infty)$ ?

 $C_2$ 

 $C\mathcal{E}$ 

 $\frac{1}{2}C\mathcal{E}$ 

 $C\mathcal{E}$ 

0

 $\frac{1}{2}C\mathcal{E}$ 

 $C\mathcal{E}$ 

 $2C\mathcal{E}$ 

 $C\mathcal{E}$ 



**Problem 12: (3 points)** Before the switched is closed, the plates of all capacitors are uncharged. A long time after the switch is closed, the charge on the left plate inside the dashed box is -5.00 nC and the potential on that plate is defined to be +7.00 V. What is the charge and potential of the indicated right plate within the dashed box?



**Problem 13:** (5 points)In the following circuit, if light bulb 2 burns out, what happens to the brightness of the other two light bulbs? Treat all the light bulbs like identical simple resistors.



**Problem 14:** (5 points) A capacitor is connected to a battery and charge is allowed to build up on the plates. Once fully charged the capacitor stores energy  $U_1$ . The capacitor is then disconnected from the battery. If the separation of the plates is then doubled, what is the energy stored on the capacitor  $U_2$ ?

(A) 
$$U_2 = \frac{1}{4}U_1$$
 (E)  $U_2 = 2U_1$   
(B)  $U_2 = \frac{1}{\sqrt{2}}U_1$  (F)  $U_2 = \sqrt{2}U_1$   
(C)  $U_2 = \frac{1}{2}U_1$  (G)  $U_2 = 4U_1$   
(D)  $U_2 = U_1$ 

**Problem 15:** (5 points) A capacitor with a dielectric,  $\kappa$ , inside is connected to a battery and charge is allowed to build up on the plates. Once fully charged there is an electric field  $E_1$  inside the capacitor. The dielectric is then removed from inside the capacitor while the battery stays connected. What is the electric field inside capacitor  $E_2$  after the dielectric is removed?

(A) 
$$E_2 = \frac{1}{\kappa^2} E_1$$
 (E)  $E_2 = \kappa E_1$   
(B)  $E_2 = \frac{1}{\sqrt{\kappa}} E_1$  (F)  $E_2 = \sqrt{\kappa} E_1$   
(C)  $E_2 = \frac{1}{\kappa} E_1$  (G)  $E_2 = \kappa^2 E_1$   
(D)  $E_2 = E_1$ 



# **Useful Constants:**

Acceleration due to gravity:  $g = 9.80 \text{ m/s}^2$ Basic unit of charge:  $e = 1.6 \times 10^{-19} \text{ C}$ Mass of electron:  $m_e = 9.11 \times 10^{-31} \text{ kg}$ Mass of proton/neutron:  $m_p = 1.67 \times 10^{-27} \text{ kg}$ Coulomb constant:  $k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$ Permittivity of free space:  $\epsilon_0 = 1/(4\pi k) = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ Permeability of free space:  $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$ Speed of light in a vacuum:  $c = 3 \times 10^8 \text{ m/s}$ Planck's Constant:  $h = 6.626 \times 10^{-34} \text{ Js}$ eV to joule conversion:  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ kilowatt-hour to joule conversion:  $1 \text{ kW} \cdot \text{hr} = 3.6 \times 10^6 \text{ J}$ Atomic Mass Unit:  $1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV/c}^2$ 

# Vector Concepts:

Unit Vector:  $\hat{r} = \frac{\vec{r}}{r}$ Gradient:  $\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$ Dot Product:  $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$ Dot Product:  $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ Cross Product:

 $|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$  $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{x} - (A_x B_z - A_z B_x) \hat{y}$  $+ (A_x B_y - A_y B_x) \hat{z}$ 

Sample Indefinite Integrals:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) + c$$

$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + c$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}} + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c(n \neq -1)$$

$$\int \frac{dx}{x} = \ln(x) + c$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{1}{x} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c$$

SI Prefixes:  $T=\times 10^{12}, G=\times 10^9, M=\times 10^6, k=\times 10^3$   $c=\times 10^{-2}, m=\times 10^{-3}$  $\mu=\times 10^{-6}, n=\times 10^{-9}, p=\times 10^{-12}, f=\times 10^{-15}$ 

# Useful integral relationships:

Spherical:  $dV = 4\pi r^2 dr$ Cylindrical (constant over r):  $dV = \pi r^2 dz$ Cylindrical (constant over z):  $dV = z2\pi r dr$ Cylindrical (with constant r):  $dA = 2\pi r dz$ Cylindrical (with constant z):  $dA = 2\pi r dr$ 

### Geometry:

Area of a Sphere:  $A = 4\pi r^2$ Volume of a Sphere:  $V = \frac{4}{3}\pi r^3$ Area of curved region of a cylinder:  $A = 2\pi rh$ Volume of a cylinder:  $V = \pi r^2 h$ 

Physics 1 Concepts: Work:  $W = \int \vec{F} \cdot d\vec{\ell}$ Kinetic Energy:  $K = \frac{1}{2}mv^2$ Momentum:  $\vec{p} = m\vec{v}$ 

# Chapter 21:

Coulomb's Law [N]:  $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$ Force due an electric field [N]:  $\vec{F} = q_0\vec{E}$ E Field Due to a pt. charge [N/C]:  $\vec{E} = \frac{kq}{r^2}\hat{r}$ E Field Due to a cont. charge dist. [N/C]:  $\vec{E} = \int \frac{kdq}{r^2}\hat{r}$ Electric dipole moment [Cm]:  $\vec{p} = q\vec{d}$ Torque on an electric dipole [Nm]:  $\vec{\tau} = \vec{p} \times \vec{E}$ Electric pot. ene. stored in electric dipole [J]:  $U = -\vec{p} \cdot \vec{E}$ 

### Chapter 22:

Electric Flux [Vm or Nm<sup>2</sup>/C]:  $\Phi_E = \int \vec{E} \cdot d\vec{A}$ 

Electric Flux when E and  $\theta$  are const. on the surface:  $\Phi_E = EA \cos \theta$ 

Gauss's Law (vacuum):  $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$ 

**Chapter 23:** The below equations generally but not always assume that  $V(\infty) = 0$  and/or  $U(\infty) = 0$ .

 $V_a - V_b = \int^b \vec{E} \cdot d\vec{\ell}$ 

Elec. pot. energy between 2 pt charges [J]:  $U = \frac{kqq_0}{r}$ Elec. pot. difference btw. two locations [V or J/C]:  $\Delta V = \frac{\Delta U}{q_0} \text{ (or often) } V = \frac{U}{q_0}$ Electric potential due to a point charge [V]:  $V = \frac{kq}{r}$ Electric potential Due to a charge dist [V]:  $V = \int \frac{kdq}{r}$ Relating  $\vec{E}$  and V:  $\vec{E} = -\vec{\nabla}V$ 

Capacitance [F]: Q = CVCapacitance for Parallel Plates [F]:  $C = \frac{\kappa \epsilon_0 A}{d}$ Energy stored in a capacitor [J]:  $U = \frac{1}{2}CV^2$ E field energy density  $[J/m^3]$ :  $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$ Definition of Dielectric Constant:  $E = \frac{E_0}{\kappa}, V = \frac{V_0}{\kappa}, C = \kappa C_0$ Eff. Cap. (series) [F]:  $\frac{1}{C_{eff}} = \sum_i \frac{1}{C_i}$ Eff. Cap. (parallel) [F]:  $C_{eff} = \sum_i C_i$ 

# Chapters 25 and 26:

Chapter 24:

Electric Current [A]:  $I = \frac{dq}{dt}$ *I* from current density [A]:  $I = \int \vec{j} \cdot d\vec{A}$ j of uniform current [A/m<sup>2</sup>]:  $|\vec{j}| = \frac{I}{A}$ j for charges in motion [A/m<sup>2</sup>]:  $\vec{j} = nq\vec{v}_d$ Ohm's Law:  $\vec{E} = \rho \vec{j}$ Ohm's Law:  $\Delta V = IR$  (or often just) V = IRResistivity and conductivity:  $\rho = \frac{1}{\sigma}$ Resistance of a wire  $[\Omega]$ :  $R = \frac{\rho \ell}{4}$ Resistance of an object [ $\Omega$ ]:  $R = \int \frac{\rho(x)dx}{A(x)}$ Power in a circuit element [W]:  $P = I\Delta V$  (or often) P = IVEff. Res. (series) [ $\Omega$ ]:  $R_{eff} = \sum R_i$ Eff. Res. (parallel) [ $\Omega$ ]:  $\frac{1}{R_{eff}} = \sum_{i} \frac{1}{R_{i}}$ Time constant for an *RC*-circuit [s]:  $\tau = RC$ Charge on a charging capacitor [C]:  $q(t) = q_{\max}(1 - e^{-t/\tau})$ Charge on a discharging capacitor [C]:  $q(t) = q_0 e^{-t/\tau}$ Current in an *RC*-circuit [A]:  $I(t) = I_0 e^{-t/\tau}$