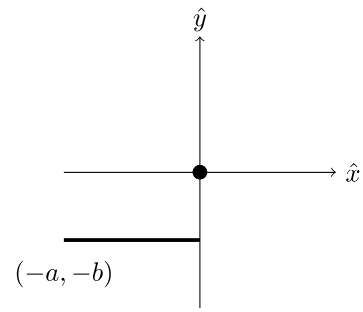
Make sure to fill out the grading sheet completely including your name, instructor, exam flavor and UIN. You are allowed to write and work on this exam copy, but your answers must be bubbled in on the grading sheet to receive credit.

Physics 207 Exam 1 – Flavor 1

Question 1: Consider two nuclei of the most common carbon isotope. Each has 6 protons and 6 neutrons, and consider that each has had its electrons "stripped" off. Initially both nuclei are at rest at a separation of 40.0 μm. They are then simultaneously released. How fast is each moving when they are infinitely far apart? (10 points)

A) 58.7 m/s
B) 76.5 m/s
C) 102 m/s
D) 144 m/s
E) 189 m/s
F) 203 m/s
G) 254 m/s
H) 328 m/s

Question 2: A thin, straight, insulating rod with uniform charge density and total charge $-Q$ has its ends at $(-a, -b)$ and $(0, -b)$. Which option below shows the integrals needed to find the electric field vector at the origin? (10 points)



A) $-\dfrac{kQb}{a}\displaystyle\int_{-a}^{0}\frac{dx}{(x^2+b^2)^{3/2}}\hat{\imath}+\frac{kQ}{a}\int_{-a}^{0}\frac{xdx}{(x^2+b^2)^{3/2}}\hat{\jmath}$

B) $-\dfrac{kQ}{a}\displaystyle\int_{-a}^{0}\frac{xdx}{(x^2+b^2)^{3/2}}\hat{\imath}+\frac{kQb}{a}\int_{-a}^{0}\frac{dx}{(x^2+b^2)^{3/2}}\hat{\jmath}$

C) $\dfrac{kQb}{a}\displaystyle\int_{-a}^{0}\frac{dx}{(x^2+b^2)^{3/2}}\hat{\imath}-\frac{kQ}{a}\int_{-a}^{0}\frac{xdx}{(x^2+b^2)^{3/2}}\hat{\jmath}$

D) $\dfrac{kQ}{a}\displaystyle\int_{-a}^{0}\frac{xdx}{(x^2+b^2)^{3/2}}\hat{\imath}-\frac{kQb}{a}\int_{-a}^{0}\frac{dx}{(x^2+b^2)^{3/2}}\hat{\jmath}$

E) $-\dfrac{kQb}{a}\displaystyle\int_{-a}^{0}\frac{dx}{(x^2+b^2)}\hat{\imath}+\frac{kQ}{a}\int_{-a}^{0}\frac{xdx}{(x^2+b^2)}\hat{\jmath}$

F) $-\dfrac{kQ}{a}\displaystyle\int_{-a}^{0}\frac{xdx}{(x^2+b^2)}\hat{\imath}+\frac{kQb}{a}\int_{-a}^{0}\frac{dx}{(x^2+b^2)}\hat{\jmath}$

G) $\dfrac{kQb}{a}\displaystyle\int_{-a}^{0}\frac{dx}{(x^2+b^2)}\hat{\imath}-\frac{kQ}{a}\int_{-a}^{0}\frac{xdx}{(x^2+b^2)}\hat{\jmath}$

H) $\dfrac{kQ}{a}\displaystyle\int_{-a}^{0}\frac{xdx}{(x^2+b^2)}\hat{\imath}-\frac{kQb}{a}\int_{-a}^{0}\frac{dx}{(x^2+b^2)}\hat{\jmath}$

Question 3: Three charges, $+2q$, $+q$ and $-3q$ are configured as shown in the figure below. What is the total potential energy of this system? (10 points)

A) $-\dfrac{7kq^2}{a}$

B) $-\dfrac{4kq^2}{a}$

C) $-\dfrac{1kq^2}{a}$

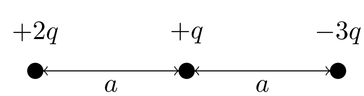
D) 0

E) $+\dfrac{3kq^2}{a}$

F) $+\dfrac{5kq^2}{a}$

G) $+\dfrac{8kq^2}{a}$

H) $+\dfrac{11kq^2}{a}$



Question 4: Two thin, insulating rods are each in the shape of 3/4 of a circle with radii a and $2a$ as shown in the figure below. The rod with radius a has a uniform charge density $+\lambda_1$ and the rod with radius $2a$ has uniform charge density $-\lambda_2$. What is the net electric potential at the origin? (10 points)

A) 0

B) $\frac{3}{8\epsilon_0}\left(\frac{\lambda_2}{2a} - \frac{\lambda_1}{a}\right)$

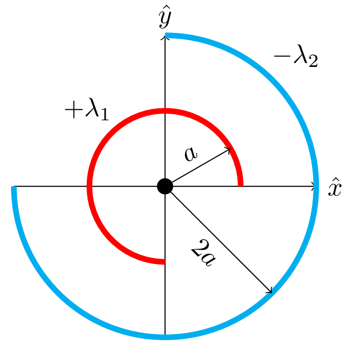
C) $\frac{3}{8\epsilon_0}\left(\frac{\lambda_1}{a} - \frac{\lambda_2}{2a}\right)$

D) $\frac{3}{8\epsilon_0}\left(\frac{\lambda_1}{a} + \frac{\lambda_2}{2a}\right)$

E) $\frac{3(\lambda_2 - \lambda_1)}{8\epsilon_0}$

F) $\frac{3(\lambda_1 - \lambda_2)}{8\epsilon_0}$

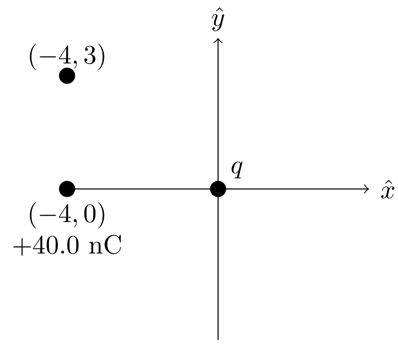
G) $\frac{3(\lambda_1 + \lambda_2)}{8\epsilon_0}$



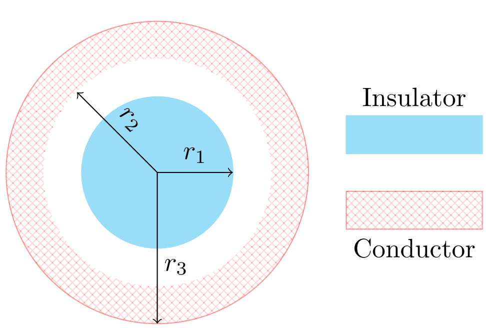
For Questions 5 and 6: There exists a +40.0 nC charge at the position (-4,0). There is an unknown charge q at the origin. What does the sign (#5) and magnitude (#6) of q have to be so that the y-component of the electric field due to only these two charges is zero at the position (-4,3)? All positions are given in m.

Question 5: (3 points)
A) Positive
B) Negative
C) Zero

Question 6: (7 points)
A) $|q| = 0.922 \times 10^{-7}$ C
B) $|q| = 1.11 \times 10^{-7}$ C
C) $|q| = 1.39 \times 10^{-7}$ C
D) $|q| = 1.85 \times 10^{-7}$ C
E) $|q| = 2.05 \times 10^{-7}$ C
F) $|q| = 2.42 \times 10^{-7}$ C



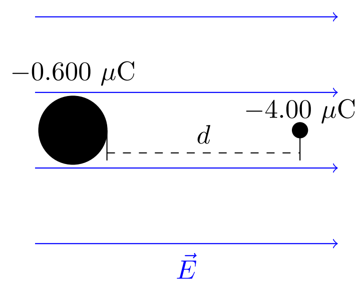
Question 7: A solid, insulating sphere with radius r_1 has a charge density $\rho(r) = \rho_0 r$ where r is measured from the center of the sphere. Surrounding the insulator is a conducting spherical shell with inner radius r_2, outer radius r_3 and total charge $-Q$. Which of the following options describes both the electric field within the insulator ($r < r_1$) and within the solid part of the conductor ($r_2 < r < r_3$)? (10 points)



Option	Insulator ($r < r_1$)	Conductor ($r_2 < r < r_3$)
A)	$\frac{\rho_0 r}{3\epsilon_0}$	0
B)	$\frac{\rho_0 r}{3\epsilon_0}$	$\frac{\rho_0 r_1}{3\epsilon_0 r^2}$
C)	$\frac{\rho_0 r}{3\epsilon_0}$	$\frac{\rho_0 r_1}{3\epsilon_0 r^2} - \frac{Q}{4\pi\epsilon_0 r^2}$
D)	$\frac{\rho_0 r^2}{4\epsilon_0}$	0
E)	$\frac{\rho_0 r^2}{4\epsilon_0}$	$\frac{\rho_0 r_1^2}{4\epsilon_0 r^2}$
F)	$\frac{\rho_0 r^2}{4\epsilon_0}$	$\frac{\rho_0 r_1^2}{4\epsilon_0 r^2} - \frac{Q}{4\pi\epsilon_0 r^2}$
G)	$\frac{4\pi\rho_0 r_1^3}{3\epsilon_0}$	0
H)	$\frac{4\pi\rho_0 r_1^3}{3\epsilon_0}$	$\frac{4\pi\rho_0 r_1^3}{3\epsilon_0} - \frac{Q}{4\pi\epsilon_0 r^2}$

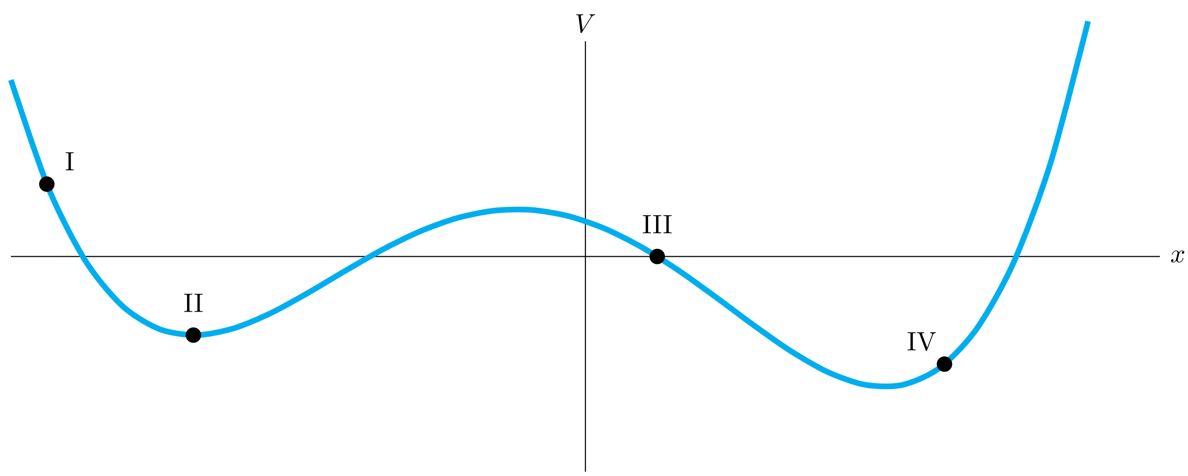
Question 8: In the figure below, object A is a solid, insulating sphere of radius 3.00 cm and net charge -0.600 μC. Object B is a point charge of -4.00 μC that is to the right of the surface of the sphere by a distance d. There is also a uniform external electric field of magnitude 250 kN/C that points to the right. If the -4.00 μC charge feels zero net electric force, what is d? (10 points)

A) 0.117 m
B) 0.147 m
C) 0.350 m
D) 0.380 m
E) 0.685 m
F) 0.715 m
G) 0.745 m
H) ∞ (An infinite distance away)



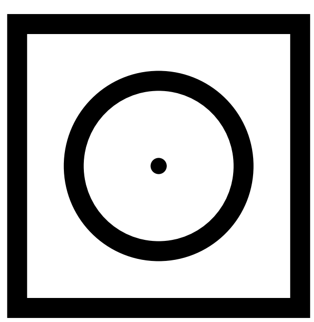
Questions 9 through 13 are 6 points each.

Question 9: The figure below represents electric potential as a function of position in the x-direction. Identify the pair of points where the electric field is zero and where the electric field points in the $-\hat{\imath}$ direction.



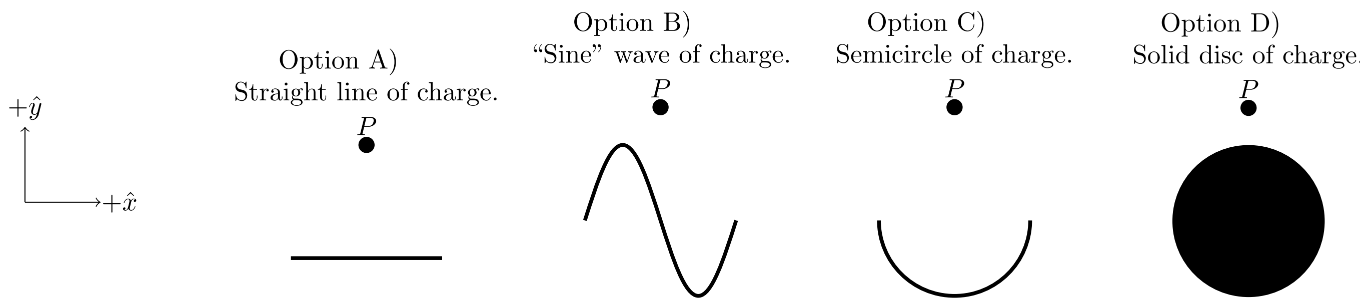
Option	$E = 0$	$\hat{E} = -\hat{\imath}$
A	II	I
B	II	III
C	II	IV
D	III	I
E	III	IV

Question 10: The picture to the right represents a point charge of $+1q$ surrounded by a conducting spherical shell with total charge $-3q$ and a conducting cubic shell with total charge $-4q$. In equilibrium, what is the total charge on the inside surface of the cubic shell?
A) $-6q$
B) $-4q$
C) $-3q$
D) $-2q$
E) $+2q$
F) $+3q$
G) $+4q$
H) $+6q$

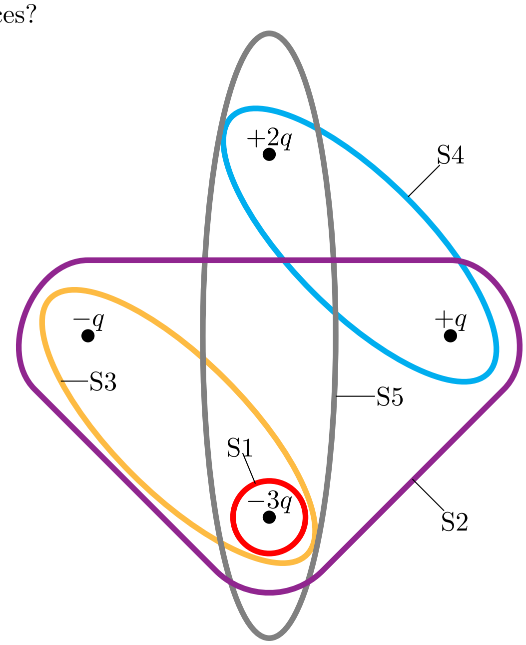


Question 11: Two charges are at points A and B. These two charges feel electric forces of the same magnitude, but the directions of the forces differ. Which of the following statements is true?
A) The two charges must have the same sign.
B) The two charges must have opposite sign.
C) The two charges can have either the same or opposite signs.

Question 12: The following picture shows four immovable and uniform positive charge distributions, and corresponding points P each centered horizontally with respect to its charge distribution. Three of the distributions give, at the points P, only an electric field along y. The remaining distribution produces an electric field with components in both x and y. Which is that remaining distribution?



Question 13: Which of the following shows the correct ranking of flux from most negative to most positive through the different possible surfaces?



A) $S5 < S4 < S3$
B) $S3 < S2 < S5$
C) $S5 < S1 < S3$
D) $S1 < S2 < S3$

Useful Constants:

Acceleration due to gravity: $g = 9.80$ m/s^2
Basic unit of charge: $e = 1.6 \times 10^{-19}$ C
Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
Mass of proton/neutron: $m_p = 1.67 \times 10^{-27}$ kg
Coulomb constant: $k = 8.99 \times 10^9$ Nm^2/C^2
Permittivity of free space: $\epsilon_0 = 1/(4\pi k) = 8.85 \times 10^{-12}$ C^2/Nm^2
Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ Tm/A
Speed of light in a vacuum: $c = 3 \times 10^8$ m/s
Planck's Constant: $h = 6.626 \times 10^{-34}$ Js
eV to joule conversion: 1 eV = 1.6×10^{-19} J
kilowatt-hour to joule conversion: 1 kW·hr=3.6×10^6 J
Atomic Mass Unit: 1 u = 1.66054×10^{-27} kg = 931.5 MeV/c^2

Vector Concepts:

Unit Vector: $\hat{r} = \frac{\vec{r}}{r}$
Gradient: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
Dot Product: $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$
Dot Product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
Cross Product:

$$\begin{aligned} |\vec{A} \times \vec{B}| =& |\vec{A}||\vec{B}| \sin\theta \\ \vec{A} \times \vec{B} =& (A_y B_z - A_z B_y)\hat{x} - (A_x B_z - A_z B_x)\hat{y} \\ &+ (A_x B_y - A_y B_x)\hat{z} \end{aligned}$$

Sample Indefinite Integrals:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) + c$$
$$\int \frac{xdx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}} + c$$
$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} + c$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c (n \neq -1)$$
$$\int \frac{dx}{x} = \ln(x) + c$$
$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{1}{x} + c$$
$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + c$$

SI Prefixes:

T=$\times 10^{12}$, G=$\times 10^9$, M=$\times 10^6$, k=$\times 10^3$
c=$\times 10^{-2}$, m=$\times 10^{-3}$
μ=$\times 10^{-6}$, n=$\times 10^{-9}$, p=$\times 10^{-12}$, f=$\times 10^{-15}$

Useful integral relationships:

Spherical: d$V = 4\pi r^2$dr
Cylindrical (constant over r): $dV = \pi r^2$dz
Cylindrical (constant over z): $dV = z 2\pi r$dr
Cylindrical (with constant r): $dA = 2\pi r$dz
Cylindrical (with constant z): $dA = 2\pi r$dr

Geometry:

Area of a Sphere: $A = 4\pi r^2$
Volume of a Sphere: $V = \frac{4}{3}\pi r^3$
Area of curved region of a cylinder: $A = 2\pi r h$
Volume of a cylinder: $V = \pi r^2 h$

Physics 1 Concepts:

Work: $W = \int \vec{F} \cdot d\vec{\ell}$
Kinetic Energy: $K = \frac{1}{2}mv^2$
Momentum: $\vec{p} = m\vec{v}$

Chapter 21:

Coulomb's Law [N]: $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$

Force due an electric field [N]: $\vec{F} = q_0\vec{E}$

E Field Due to a pt. charge [N/C]: $\vec{E} = \frac{kq}{r^2}\hat{r}$

E Field Due to a cont. charge dist. [N/C]:
$\vec{E} = \int \frac{kdq}{r^2}\hat{r}$

Electric dipole moment [Cm]: $\vec{p} = q\vec{d}$

Torque on an electric dipole [Nm]: $\vec{\tau} = \vec{p} \times \vec{E}$

Electric pot. ene. stored in electric dipole [J]:
$U = -\vec{p} \cdot \vec{E}$

Chapter 22:

Electric Flux [Vm or Nm^2/C]: $\Phi_E = \int \vec{E} \cdot d\vec{A}$

Electric Flux when E and θ are const.
on the surface: $\Phi_E = EA\cos\theta$

Gauss's Law (vacuum): $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$

Chapter 23: The below equations generally but not always assume that $V(\infty) = 0$ and/or $U(\infty) = 0$.

Elec. pot. energy between 2 pt charges [J]:
$U = \frac{kqq_0}{r}$

Elec. pot. difference btw. two locations [V or J/C]:
$\Delta V = \frac{\Delta U}{q_0}$ (or often) $V = \frac{U}{q_0}$

Electric potential due to a point charge [V]:
$V = \frac{kq}{r}$

Electric potential Due to a charge dist [V]:
$V = \int \frac{kdq}{r}$

Relating $\vec{E}$ and V: $\vec{E} = -\vec{\nabla}V$

$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{\ell}$