A.

The electric potential as a function of coordinate x is plotted below. Answer the following making sure to give a reason for your choice.

(i) At which point is the magnitude of the electric field the greatest?

Largest slope at Location B

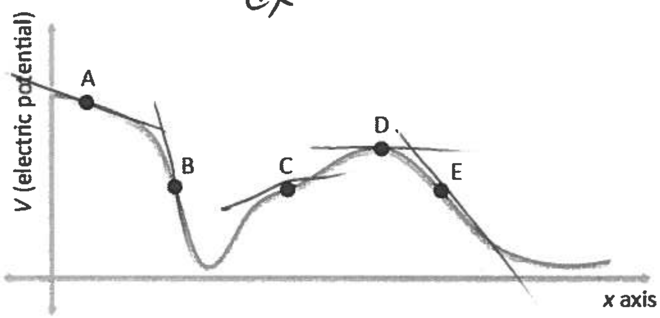
(ii) At which point(s) is the electric field directed along positive x axis?

At points A, B, E slope is negative, E positive

(iii) At which point(s) is the electric field directed along the negative x axis?

At point C slope is positive so E is negative

NOTE: $E_x = -\frac{dV}{dx}$



LO	P	F
5.1		
7.1		
25.1		
5.2		
7.2		
25.2		
5.3		
7.3		
25.3		

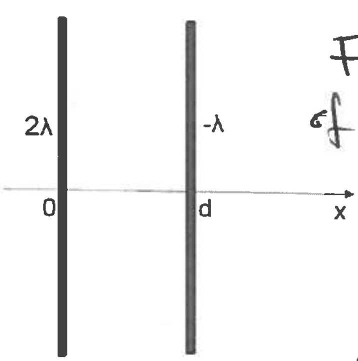
B.

Two infinite lines have the linear charge densities 2λ and – λ as shown in a figure. They are separated by a distance d. At which position along x axis is the net electric field produced by these line charges equal to zero?

$\vec{E}_T = \vec{E}_1 + \vec{E}_2$ with $|E_1| = \frac{2\lambda}{2\pi\epsilon_0 r_1}$

& $|E_2| = \frac{-\lambda}{2\pi\epsilon_0 r_2}$

For $\vec{E}_T = 0$ the directions of the 2 fields must be opposite.



$$\frac{2\lambda}{2\pi\epsilon_0 x} = \frac{\lambda}{2\pi\epsilon_0 (x-d)}$$

only possible to the right of the −λ line charge.

$2(x-d) = x$ so, Solving for $x = 2d$

LO	P	F
2.1		
3.1		
5.4		
12.1		
18.1		

C.

A proton (charge +e) has an initial kinetic energy *E* very far from a stationary Krypton nucleus (charge +36e). If the proton approaches the Krypton nucleus head-on, how close does it come before reversing its direction of motion (answer in terms of *E,* the given charges and other known constants? Assume that the Krypton nucleus remains stationary during the process.

LO	P	F
3.2		
5.5		
20.1		
21.1		

Energy is conserved so

$$U + KE)_{\text{start at } \infty} = U + KE)_{\text{stopping}}$$

so $$E = \frac{kqQ}{r_0} = \frac{36ke^2}{r_0}$$

solving for $$r_0 = \frac{36ke^2}{E}$$

Problem 1

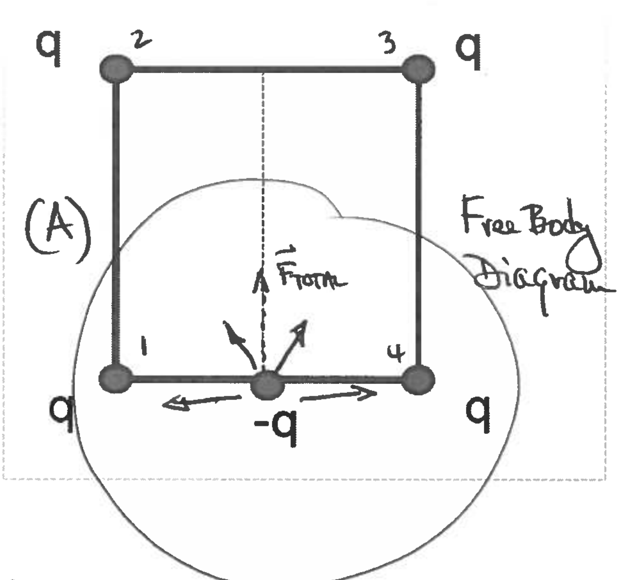
Four identical point charges, q, are fixed at each corner of a square insulating frame of side *a*. A charge –q is placed in the middle of the bottom side (see Figure).

A. Draw a free body diagram for the –q charge.

B. Find the magnitude and direction of the net electric force produced by the four positive charges on the charge –q.

C. Find the work done by the set of four positive charges when bringing the –q charge from infinity to its current position.

D. Find the work done by the set of four positive charges when moving the charge –q from the middle of the bottom side to the middle of the top side.



(B) $\vec{F}_{-q} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

from symmetry $\vec{F}_1 = -\vec{F}_4$ and

$F_{2x} = -F_{3x}$ leaving

$$\vec{F}_{-q} = (F_{2y} + F_{3y})\hat{j}$$

$$\vec{F}_{-q} = 2\left[\frac{kq^2}{(\frac{a}{2})^2+a^2}\left(\frac{a}{((\frac{a}{2})^2+a^2)^{1/2}}\right)\right]\hat{j}$$

$$= \frac{2kaq^2}{((\frac{a}{2})^2+a^2)^{3/2}}\hat{j}$$

LQ	P	F
1.1		
8.1		
1.2		
8.2		
1.3		
8.3		
1.4		
8.4		
2.2		
9.1		
20.2		
21.2		
20.3		
21.3		
23.1		

(C) $$W_{coulomb} = -\Delta U = -\left[U_{final} - \overset{0}{\underset{@\, r=\infty}{U_{init}}}\right]$$

$$= -\left(-q\left[\frac{kq}{(\frac{a}{2})} + \frac{kq}{((\frac{a}{2})^2+a^2)^{1/2}} + \frac{kq}{((\frac{a}{2})^2+a^2)^{1/2}} + \frac{kq}{\frac{a}{2}}\right]\right)$$

$$= 2kq^2\left[\frac{1}{(a/2)} + \frac{1}{((\frac{a}{2})^2+a^2)^{1/2}}\right]$$

(D) Work done by coulomb force moving charge across the square

$W_{coulomb} = q_{moved}\,\Delta V$ but V at two locations are the same so no work is done.

Problem 2

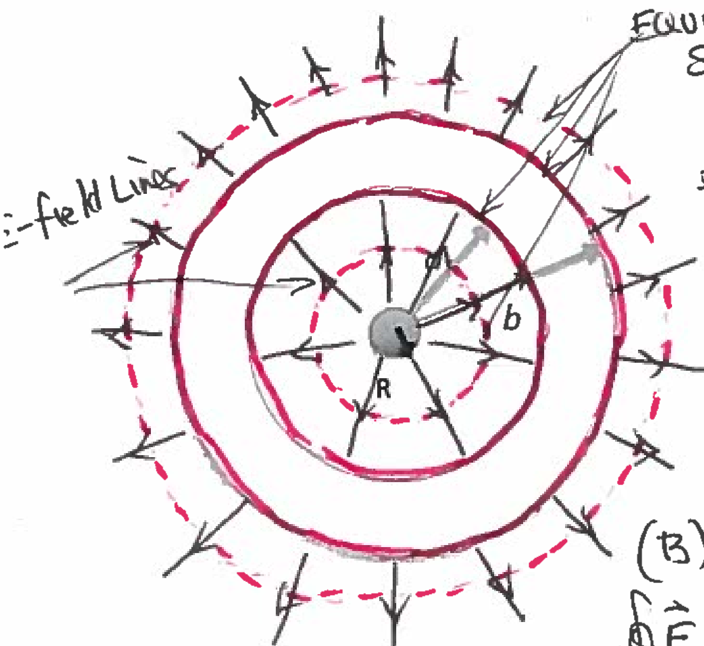
A total positive charge Q is distributed uniformly on a **solid conducting sphere** of radius R. A **conducting spherical shell** with net charge $+3Q$ has an inner radius $a > R$, outer radius b, and is concentric to the inner sphere.

A. How much charge is located on the inner and outer surfaces of the conducting spherical shell?

B. Find the electric field in the regions, $0 < r < R$; $R < r < a$; $a < r < b$; $b < r$.

C. Taking the potential at infinity to be 0, calculate the potential difference $V(b) - V(R)$.

D. Sketch the electric field lines for this configuration. On this same sketch, indicate 3-4 different equipotential surfaces.



(A) for Gaussian surface at $r \sim a$ charge enclosed must be 0 since $E_{cond} = 0$. So $-Q$ on inner surface. ($r = a$) For outer surface at $r = b$ $4Q$ is on the surface.

(B) Using Gauss's Law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\epsilon_0}$ in the

LO	P	F
5.6		
16.1		
19.1		
16.2		
19.2		
18.2		
19.3		
18.3		
19.4		
26.1		
6.1		
13.1		
27.1		
24.1		

four regions: $r < R$ $E = 0$ inside a conductor

$R < r < a$ $E(r) = \frac{KQ}{r^2}$ radially out

$a < r < b$ $E = 0$ again inside a conductor.

$r > b$ $E(r) = \frac{4KQ}{r^2}$ radially out.

(C) Using $V(b) - V(R) = -\int_R^b \vec{E} \cdot d\vec{r} = -\int_R^a \vec{E} \cdot d\vec{r} = -\int_R^a \frac{KQ}{r^2} dr$

$= \left. \frac{KQ}{r} \right|_R^a = \frac{KQ}{a} - \frac{KQ}{R}$

$= \frac{KQ(R-a)}{Ra}$

Problem 3

A positive electric charge Q is distributed uniformly around a thin ring of radius a. The ring is positioned with its center fixed at the origin of the coordinate system and with the x axis perpendicular to the plane of the ring. A point positive charge q of mass m is placed at a point P on the ring axis at a distance x from the center of the ring. Then it is released from rest.

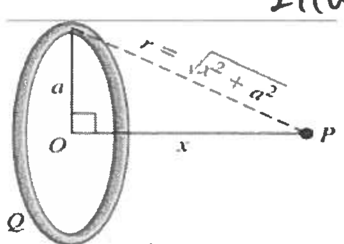
A. Find the potential produced by the ring at a point P.

B. Find the electric field produced by the ring at a point P.

C. Find the electric potential energy stored in the ring-point charge system.

D. Find the maximum speed acquired by the charge q after its release from rest.

note $\lambda = \frac{Q}{2\pi a}$



(A) for continuous charge distributions we know $V(P) = \int \frac{k\,dq'}{|\vec{r}-\vec{r}'|} = \int \frac{k\lambda a\,d\phi'}{(a^2+x^2)^{1/2}}$

so $V(x) = \frac{k\lambda a\,(2\pi)}{(a^2+x^2)^{1/2}} = \frac{kQ}{(a^2+x^2)^{1/2}}$

(B) on the axis only E component along x so,

$$E_x(x) = -\frac{\partial}{\partial x}V(x) = -kQ\left[-\tfrac{1}{2}(a^2+x^2)^{-3/2}[2x]\right]$$

(c) Energy stored when bringing in charge q from infinity $U = q\,\Delta V = q\left[\frac{kQ}{(a^2+x^2)^{1/2}}\right]$

LO	P	F
5.7		
7.4		
24.2		
7.5		
25.4		
5.8		
20.4		
3.3		
21.4		

(d) For conservation of energy

$$(U+KE)_{start} = (U+KE)_{at\ \infty}$$

$$\frac{qQ}{(a^2+x^2)^{1/2}} = \frac{1}{2}mv^2$$

Solving for $v = \sqrt{\frac{2qQ}{m(a^2+x^2)^{1/2}}}$