# Physics 208 - Exam I

Spring 2019 (513-517; 520-524)  February 11, 2019.

Solutions

Please fill out the information and read the instructions below, but
**do not open the exam** until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs.) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 6 numbered (3 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!
5. You will be allowed to use only non-programmable calculators on this exam.
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be quite distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

**Fill out the information below and sign to indicate your understanding of the above rules**

**Name:** ______________________  **UIN:** ______________________

**Signature:** ______________________  **Section Number:** ________________

**Instructor:** **Ross** **Webb**

(circle one)

**A)** A point charge 2Q is placed at the origin and a point charge –Q is placed at position +D on the x-axis.

**(a)** At what point on the x-axis is the electric field due to these charges equal to zero?

**(b)** At what point on the x-axis is the electric potential due to these charges equal to zero?

**(c)** If another point charge equal to -2Q is brought from infinitely far away, and placed at the point identified in part (b), what is the change of the total potential energy for the system in the process and why?


a) $\vec{E}_{TOTAL} = \vec{E}_{2Q}(x) + \vec{E}_{-Q}(x)$

$= \frac{2kQ}{x^2}\hat{i} - \frac{kQ}{(x-D)^2}\hat{i}$ for x to the right of $-Q$

Setting this to zero gives $\frac{2kQ}{x^2} - \frac{kQ}{(x-D)^2} = 0$ or $2(x-D)^2 = x^2$

Solving the quadratic $x^2 - 4xD - 2D^2 = 0$

$x = \frac{+4D \pm \sqrt{16D^2 - 8D^2}}{2} = (2 \pm \sqrt{2})D$

only $(2+\sqrt{2})D$ is to the right of $-Q$.

| LO | P | F |
|---|---|---|
| 2.1 | | |
| 3.1 | | |
| 5.1 | | |
| 10.1 | | |
| 11.1 | | |
| 3.2 | | |
| 5.2 | | |
| 23.1 | | |
| 20.1 | | |
| 22.1 | | |
| | | |

b) $V_{TOTAL} = \frac{2kQ}{x} - \frac{kQ}{(x-D)} = 0$ to right of $-Q$

$\frac{2kQ}{x} - \frac{kQ}{(D-x)} = 0$ to left of $-Q$

Solving Right $2(x-D) = x$ $x = 2D$

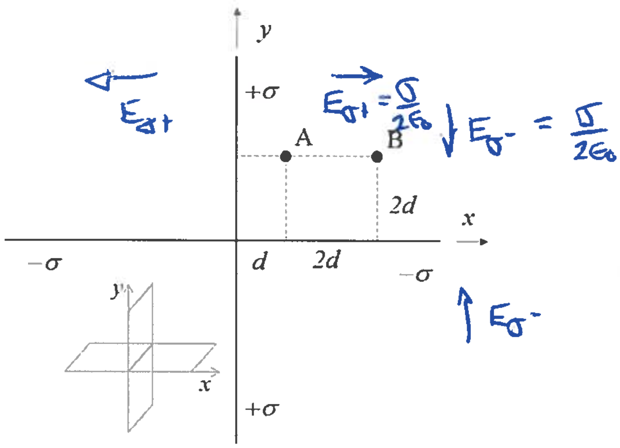
Left $2(D-x) = x$ $x = \frac{2}{3}D$

c) Change in the potential energy of the system of charges

$\Delta U = -2Q[\Delta V]$ where $\Delta V = V_{final} - V_{\infty}$ with $V_{final} = 0$ & $V_{\infty} = 0$

So $\Delta U = 0$ we do no work bringing this charge in from infinity to this location.

**B)** Two infinite planes of charge, the one with $+\sigma$ uniform charge density sitting on the y-z plane, and one with $-\sigma$ sitting on the x-z plane, meet along the z axis as shown. See also the inset perspective sketch.




**(a)** Find the electric field (magnitude and direction) at the point A [coordinates (d, 2d, 0)] as shown.

**(b)** Find the difference in electric potential energy between the two points A and B, e.g. find $\Delta V = V_B - V_A$.

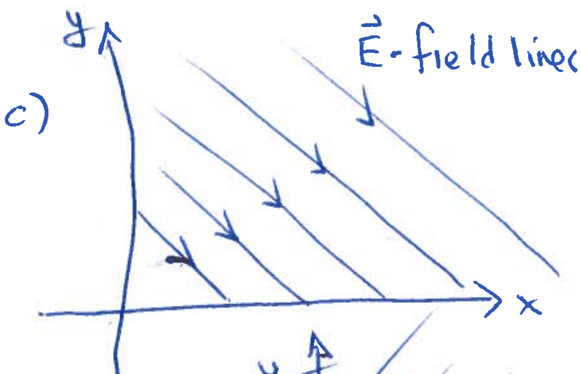
**(c)** In two separate sketches, 1) sketch 5 field lines and 2) cross-sections of 5 equipotential surfaces, as they appear in the first quadrant of the x-y plane.

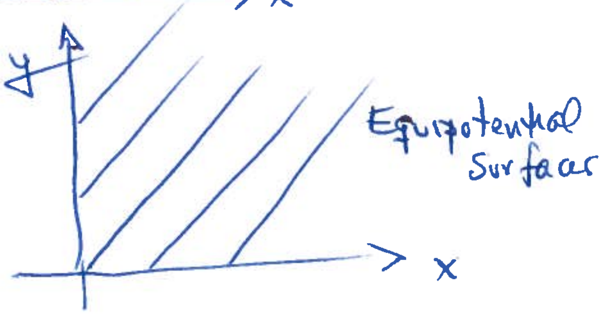
a) $\vec{E}_{TOTAL}(d, 2d, 0) = \vec{E}_{\sigma+} + \vec{E}_{\sigma-} = \frac{\sigma}{2\epsilon_0}\hat{i} - \frac{\sigma}{2\epsilon_0}\hat{j}$ in first quadrant

$= \frac{\sigma}{2\epsilon_0}(\hat{i} - \hat{j})$

b) $\Delta V = -\int_A^B \vec{E}_T \cdot d\ell = -\left(\vec{E}_T \cdot (2d\hat{i})\right) = -\frac{2d\sigma}{2\epsilon_0}$

c)



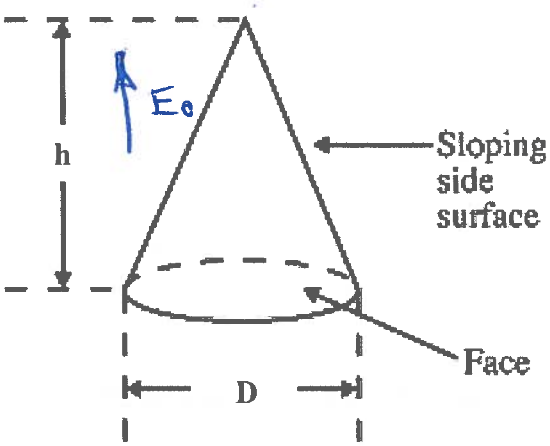




| LO | P | F |
|---|---|---|
| 1.1 | | |
| 2.2 | | |
| 3.3 | | |
| 11.2 | OMITTED | |
| 12.1 | | |
| 2.3 | | |
| 3.4 | | |
| 7.1 | | |
| 24.1 | OMITTED | |
| 26.1 | | |
| 13.1 | | |
| 27.1 | | |
| | | |

**C)** A cone is resting on a tabletop as shown in the figure with its bottom face in the horizontal plane. A uniform electric field of magnitude $E_o$ N/C points vertically upward, normal to this plane.

**(a)** How much charge is contained within the volume of the cone?

**(b)** How much electric flux passes through the sloping side surface area of the cone?




a) All E-field lines enter the bottom surface of the cone and exit the side. Hence,

$$\oint_{cone} \vec{E}\cdot d\vec{A} = 0 \quad \& \quad q_{en} = 0\,!$$

b) From part A) the total flux through the cone is zero!

$$\oint_{cone} \vec{E}\cdot d\vec{A} = \int_{Bottom} \vec{E}\cdot dA + \int_{side} \vec{E}\cdot d\vec{A} = 0$$

then

$$\int_{Bottom} \vec{E}\cdot dA = -\int_{side} E\cdot dA$$

find

$$\int_{Bottom} \vec{E}\cdot dA = \int (E_0\,\hat{j})\, dA(-\hat{j})$$

$$= -\frac{E_0\,\pi D^2}{4}$$

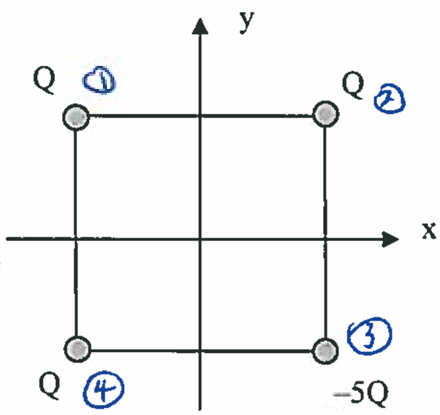
so

$$\int_{side} \vec{E}\cdot d\vec{A} = +\frac{E_0\,\pi D^2}{4}$$

| LO | P | F |
|---|---|---|
| 13.2 | | |
| 15.1 | | |
| 16.1 | | |
| 3.5 | | |
| 15.2 | | |
| 16.2 | | |
| | | |

**Problem 1.** The four charges are located at locations given by the $x$-$y$ coordinates $(d, d)$, $(d, -d)$, $(-d, d)$, $(-d, -d)$.

**(a)** What is the electric field at the origin (as components)?




$\vec{E}_T(0,0) = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

$\vec{E}_1 = \frac{KQ}{2d^2}(\cos 45°\hat{i} - \sin 45\hat{j})$; $\vec{E}_2 = \frac{KQ}{2d^2}(-\cos 45°\hat{i} - \sin 4$

$\vec{E}_3 = \frac{5KQ}{2d^2}(\cos 45\hat{i} - \sin 45\hat{j})$; $\vec{E}_4 = \frac{KQ}{2d^2}(\cos 45°\hat{i} + \sin 45\hat{j})$

$\vec{E}_T = \frac{KQ}{2d^2}(6\cos 45°)\hat{i} + \frac{KQ}{2d^2}(-6\sin 45°)\hat{j}$

**(b)** Find the electric potential, at a position $(d, 0)$. (That is, on the x axis and midway between the charges $Q$ and $-5Q$ on the right.) As a reference, assume zero potential at infinity.

**(c)** Very far away from all of these charges, would you expect the electric field lines to be pointing away from them, or towards them? Explain your reasoning.

**(d)** Find the force on the charge Q at position (-d, -d) due to the other three charges.

**(e)** Find the change in the potential energy of this system of charges if the -5Q charge is moved from its current location to a point at infinity.

b) $V(d,0) = V_1(d,0) + V_2(d,0) + V_3(d,0) + V_4(d,0)$

$= \frac{kq}{\sqrt{5}d} + \frac{kq}{d} + \frac{-5kq}{d} + \frac{kq}{\sqrt{5}d}$

$= \frac{2kq}{d}\left(\frac{1}{\sqrt{5}} - 2\right)$

c) Very far away the collection of charges looks like a single point charge of total charge $-2Q$. As a result the field lines will be pointing towards the origin.

d) $\vec{F}_{Q\text{ at }(-d,-d)} = \frac{5KQ^2}{4d^2}(+\hat{i}) + \frac{KQ^2}{4d^2}(-\hat{j}) + \frac{KQ^2}{8d^2}(-.707\hat{i} - .707\hat{j})$

e) $\Delta U = q\Delta V = -5Q\Delta V = -5Q\left[0 - \left(\frac{2KQ}{2d} + \frac{KQ}{2\sqrt{2}d}\right)\right]$

$= 5Q\left(\frac{2KQ}{2d} + \frac{KQ}{2\sqrt{2}d}\right)$

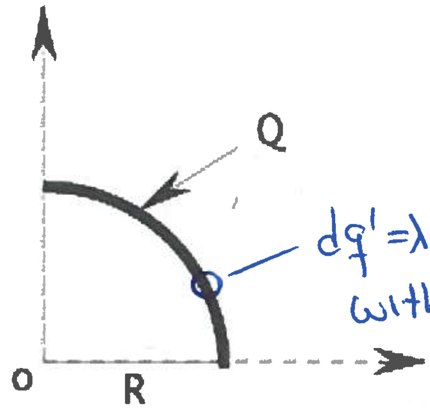
| LO | P | F |
|---|---|---|
| 1.2 | | |
| 2.4 | | |
| 10.2 | | |
| 11.3 | | |
| 23.2 | | |
| 13.3 | | |
| 1.3 | | |
| 2.5 | | |
| 8.1 | | |
| 9.1 | | |
| 20.2 | | |
| | | |

**Problem 2.** Find the electric field at the origin due to a 90° arc formed by a uniform line charge. The radius of the arc is $R$, and a total charge $+Q$ is distributed uniformly along its length. [To get full credit for this problem you will need to show all the steps you have gone through to arrive at your answer.]

$$\vec{E}(\text{origin}) = \int \frac{k\,dq'}{|\vec{r}-\vec{r}\,'|^2}\left(\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\right)$$

$\vec{r} = 0$ & $\vec{r}\,' = R\cos\theta'\,\hat{\imath} + R\sin\theta'\,\hat{\jmath}$

& $|\vec{r}-\vec{r}\,'|^2 = R^2$




$dq' = \lambda R\, d\theta'$

with $\lambda = \frac{Q}{(\pi R/2)} = \frac{2Q}{\pi R}$

putting this all together

$$\vec{E}(0,0) = \int \frac{k\left(\frac{2Q}{\pi R}\right) R\, d\theta'}{R^2}\left[\frac{-R\cos\theta'\,\hat{\imath} - R\sin\theta'\,\hat{\jmath}}{R}\right]$$

$$= \frac{k\left(\frac{2Q}{\pi R}\right)}{R}\left[\underbrace{\int_0^{\pi/2} -\cos\theta'\, d\theta'}_{=-1}\,\hat{\imath} + \underbrace{\int_0^{\pi/2} -\sin\theta'\, d\theta'}_{=-1}\,\hat{\jmath}\right]$$

$$= \frac{-k\left(\frac{2Q}{\pi R}\right)}{R}\left(\hat{\imath} + \hat{\jmath}\right)$$

$$= \frac{k\lambda}{R}\left(-\hat{\imath} - \hat{\jmath}\right)$$

| LO | P | F |
|---|---|---|
| 1.4 | | |
| 1.5 | | |
| 2.6 | | |
| 5.3 | | |
| 7.2 | | |
| 10.3 | | |
| 12.2 | | |
| | | |

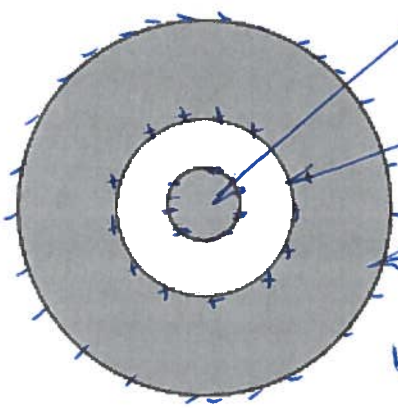
**Problem 3.** The figure shows a cross-section view of a very long cylindrical cable. There is an outer tube made of copper, inner radius $2R$, outer radius $4R$. The inner copper wire has radius $R$ and is concentric with the tube. The inner wire has charge density $-2\lambda$ (per unit length), while the outer tube carries total charge $+\lambda$ (per unit length). Answer the following in terms of known constants and the constants given. [Once again, you need to show solution method to get full credit on this problem.]

**(a)** How does the charge arrange itself on the cable's surfaces at $r = R$, $2R$ and $4R$? Explain your reasoning.

**(b)** Find the electric field at a distance $r$ from the center of the wire, in the gap region, $R < r < 2R$. What is its direction?

**(c)** What is $E$ a distance $r$ outside the tube ($r > 4R$)?

**(d)** What will be the voltage difference between points at radius $R$ and $4R$ for this system? $\Delta V = V_{4R} - V_R$.



a) On the inner conducting cylinder the $-2\lambda$ charge is on the surface at $r = R$

On the inner surface of the outer cylinder a charge of $+2\lambda$ will appear to cancel the effect of the inner cylinder in the conductor. $+2\lambda$ at $r = 2R$.

The outer surface will have the remaining charge on the outer cylinder $\lambda - 2\lambda = -\lambda$ at $R = 4R$.

b) Using Gauss's law find $\vec{E}(r)$ for $R<r<2R$

$\oint \vec{E}\cdot d\vec{A} = q_{en}/\epsilon_0$ So $E(r)(2\pi r \ell) = -\dfrac{2\lambda\ell}{\epsilon_0}$ ; $E(r) = \dfrac{-2\lambda}{2\pi\epsilon_0 r}$ (inward)

c) Find $\vec{E}(r)$ outside the tube $\oint \vec{E}\cdot d\vec{A} = E(r)(2\pi r\ell) = \dfrac{-\lambda\ell}{\epsilon_0}$

solving $E(r) = \dfrac{-\lambda}{2\pi\epsilon_0 r}$ (again inward)

d) $\Delta V = V_{4R} - V_R = -\int_R^{4R} \vec{E}\cdot d\vec{\ell} = -\int_R^{2R} + -\int_{2R}^{4R}$

$= -\int_R^{2R} \dfrac{-2\lambda}{2\pi\epsilon_0 r}\,dr$

$= \dfrac{2\lambda}{2\pi\epsilon_0} \cdot \ln r \Big|_R^{2R} = \dfrac{\lambda}{\pi\epsilon_0}\ln(2)$

$\parallel$ 0 since it is in the conductor & $\vec{E} = 0$!

| LO | P | F |
|---|---|---|
| 18.1 | | |
| 19.1 | | |
| 12.3 | | |
| 15.3 | | |
| 16.3 | | |
| 18.2 | | |
| 12.4 | | |
| 15.4 | | |
| 18.3 | | |
| 7.3 | | |
| 19.2 | | |
| 26.2 | | |
| | | |