# Physics 208 - Exam I

Fall 2017 (all sections)   September 25th, 2017.

Please fill out the information and read the instructions below, but
**do not open the exam** until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 6 numbered (3 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!
5. You will not be allowed to use calculators on this exam since all problems use symbols in their problem statements or the numbers have been chosen to make any required arithmetic calculations straightforward. If there are problems resulting in numerical answers you may leave them in fractional form.
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be quite distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

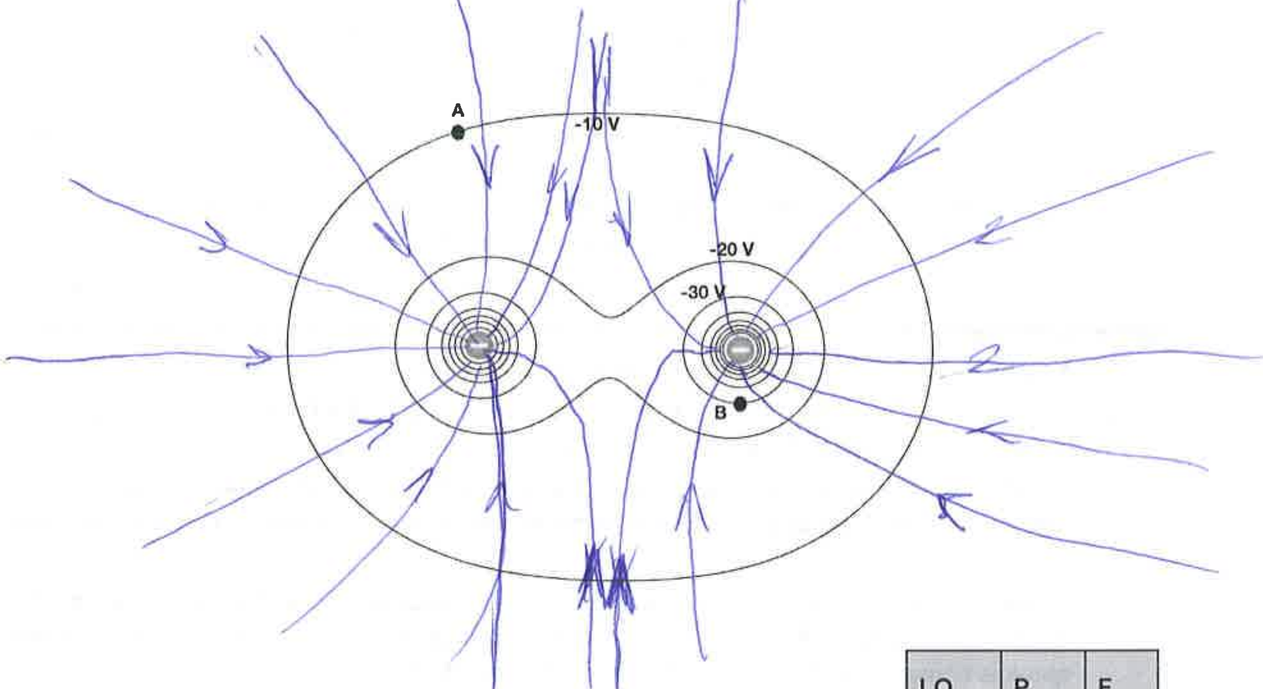
**Fill out the information below and sign to indicate your understanding of the above rules**

Name: Key   UIN: ____________
(printed legibly)

Signature: ______________________   Section Number: __________

Instructor: Mioduszewski   Kocharovskaya   Saslow
**(circle one)**

A. A system of two identical negative point charges is shown in the figure below together with equipotential surfaces. **Draw electric field lines for this system**. What is the work done by the fields moving a small positive charge of 0.1 nC from point A to point B?

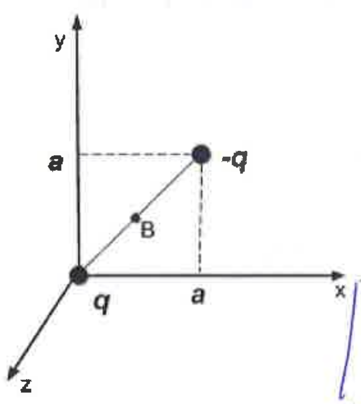



| LO | P | F |
|---|---|---|
| 13.1 | | |
| 27.1 | | |

$$W = -\Delta U = -(U_B - U_A)$$

$$\boxed{W = q(V_A - V_B)} = 0.1\cdot 10^{-9}(-10+30) = 2\cdot 10^{-9}\,J = \boxed{2\,nJ}$$

B. What is the torque (with respect the point B, which is the center of the dipole) exerted on a system of two point charges ***q*** and ***-q*** connected by a non-conducting rod due to a uniform electric field $\vec{E} = E_x\hat{i}$ if the positive charge ***q*** is at the origin and the negative charge ***-q*** is at (x,y) = ***(a,a)***? (Make sure to provide torque's magnitude and direction. Express your answer in terms of known values ***q***, $\boldsymbol{E_x}$ and ***a***.)




$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\vec{p} = q(-a\hat{i} - a\hat{j})$$

$$\vec{\tau} = q(-a)(\hat{i}+\hat{j}) \times E_x\hat{i}$$

$$\boxed{\vec{\tau} = q a E_x \hat{k}}$$

| LO | P | F |
|---|---|---|
| 2.1 | | |
| 14.1 | | |

C. Two identical charged particles are found at rest at distance ***L*** from each other at some initial time. Particle's charge ***q*** is known. What is the final kinetic energy of each charged particle after a very long time? (Neglect effects of gravity and assume that there is nothing else in the Universe except for these two charged particles. Express your answer in term of known values ***q*** and ***L***.)

$E = const$

$K_i + U_i = K_f + U_f$ ; $K_i = 0$; $U_f = 0$; $K_f = K_1 + K_2$

$U_i = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L}$ ; $K_1 = K_2$

$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{L} = 2K_1$

$$K_1 = K_2 = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{L}$$

| LO | P | F |
|---|---|---|
| 5.1 | | |
| 20.1 | | |
| 21.1 | | |

D. Electric field in a region of space is given by $\vec{E} = \boldsymbol{C} y^2 \hat{j}$ . What is the electric potential difference $V_1 - V_2$ between a point 1 $(x_1,y_1)$=***(0,a)*** and a point 2 $(x_2,y_2)$=***(0,b)***? (***C*** is a known constant that has units of V/m$^3$. Express your answer in terms of known values ***C***, ***a*** and ***b***.)

$V_1 - V_2 = \int_1^2 \vec{E} \cdot d\vec{l}$

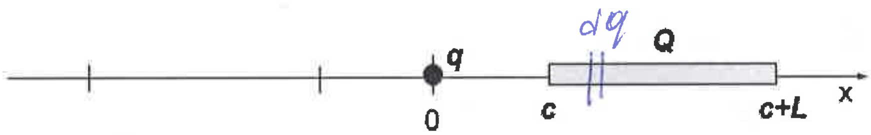
$V_1 - V_2 = \int_a^b C y^2 \hat{j}\, dy \cdot \hat{j} = \frac{C}{3}(b^3 - a^3)$

$$V_1 - V_2 = \frac{C}{3}(b^3 - a^3)$$

| LO | P | F |
|---|---|---|
| 7.1 | | |
| 26.1 | | |

## Problem I.

A point charge ***q*** (positive) is fixed at the origin. Another charge ***Q*** (positive) is uniformly distributed along the x-axis from x=***c*** to x=***c+L***. (Express all answers in terms of known values ***q, Q, c,*** and ***L***.)




A. What is the electric field (magnitude and direction) produced by the uniformly distributed charge ***Q*** at the position of the point charge ***q*** (at the origin)?

$$d\vec{E} = (-\hat{i})\frac{dq}{x^2}\frac{1}{4\pi\varepsilon_0} \ ; \quad \vec{E} = \int_c^{c+L} d\vec{E} \ ; \quad dq = \lambda dx = \frac{Q}{L}dx$$

$$\vec{E} = \int_c^{c+L} \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\frac{dx}{x^2}(-\hat{i}) = -\hat{i}\frac{Q}{4\pi\varepsilon_0 L}(-1)\left(\frac{1}{c+L}-\frac{1}{c}\right) = \boxed{-\hat{i}\frac{Q}{4\pi\varepsilon_0}\frac{1}{(c+L)c}}$$

B. What is the force (magnitude and direction) exerted by the uniformly distributed charge ***Q*** on the point charge ***q***?

$$\vec{F} = q\vec{E} \ ; \quad \boxed{\vec{F} = -\hat{i}\frac{Qq}{4\pi\varepsilon_0}\frac{1}{(c+L)c}}$$

C. What would be the force exerted on the point charge ***q*** if the uniformly distributed charge is replaced by a point charge ***Q*** located on the x-axis at x=***c+L/2***? Compare the result to the force found in A. State clearly in which case (A or B) the force is greater and show your reasoning mathematically.

$$\boxed{\vec{F}_p = \frac{-\hat{i}}{4\pi\varepsilon_0}\frac{Qq}{(c+\frac{L}{2})^2}}$$

$$(c+L)c = c^2 + cL$$

$$(c+\tfrac{L}{2})^2 = c^2 + cL + \frac{L^2}{4}$$

for any $L>0$ $\quad (c+\frac{L}{2})^2 > (c+L)c$

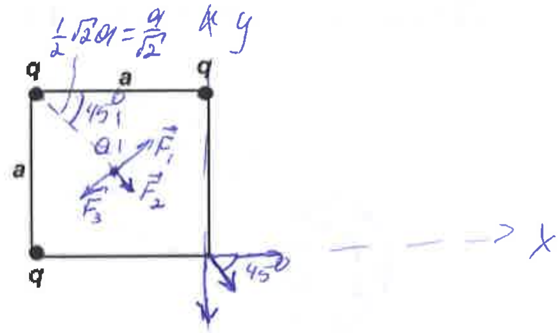
therefore

$$\boxed{|\vec{F}_p| < |\vec{F}|}$$

| LO | P | F |
|---|---|---|
| 5.2 | | |
| 6.1 | | |
| 7.2 | | |
| 8.1 | | |
| 8.2 | | |
| 12.1 | | |

## Problem II.

Three identical positive point charges ***q*** are fixed at three corners of a square with sides of length ***a***, as shown in the figure. (Express all answers in terms of known values ***q, Q*** and ***a***.)




A. What is the electric potential energy of the system?

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{a} + \frac{q^2}{a} + \frac{q^2}{\sqrt{a^2+a^2}}\right) = \boxed{\frac{1}{4\pi\varepsilon_0} q^2\left(\frac{2}{a} + \frac{1}{\sqrt{2}a}\right)}$$

B. What is the electric field (direction and magnitude) produced by this system of three charges at the fourth (empty) corner of the square?

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q}{a^2}\hat{i} + \frac{q}{a^2}(-\hat{j}) + \frac{q}{2a^2}(\hat{i}\cos 45^\circ - \hat{j}\sin 45^\circ)\right)$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left(1 + \frac{\sqrt{2}}{4}\right)(\hat{i} - \hat{j})}$$

C. What is the force (direction and magnitude) exerted by the three positive point charges ***q*** located at the corners of the square on the fourth positive charge ***Q*** positioned at the **center** of the square?

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{F}_1 = -\vec{F}_3;$$

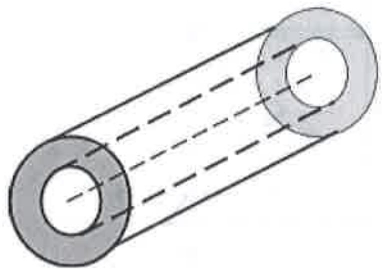
$$\vec{F}_{net} = \vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{\left(\frac{a}{\sqrt{2}}\right)^2}(\hat{i}\cos 45^\circ - \hat{j}\sin 45^\circ)$$

$$\boxed{\vec{F}_{net} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2}(\hat{i} - \hat{j})}$$

| LO | P | F |
|---|---|---|
| 1.1 | | |
| 2.2 | | |
| 2.3 | | |
| 5.3 | | |
| 9.1 | | |
| 10.1 | | |
| 11.1 | | |
| 20.2 | | |

## Problem III.

An uncharged infinitely long hollow conducting cylinder with inner radius $R_1$ and outer radius $R_2$ has a very thin charged insulator string with uniform linear charge density $\Lambda_o$ at its axis. (Express all answers in terms of known values $R_1$, $R_2$, and $\Lambda_o$.)

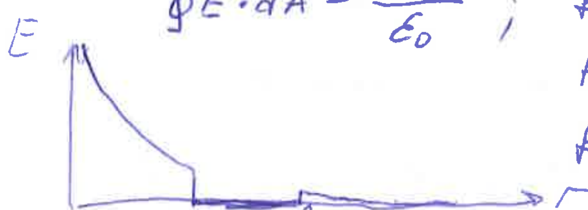


A. What is the electric field as a function of distance from the axis of the cylinder? Make sure to consider all relevant regions, r < $R_1$, $R_1$<r<$R_2$ and r>$R_2$. Draw a graph of electric field magnitude as a function of distance from the axis of the cylinder.

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ ; for $r<R_1$ $E 2\pi r z = \frac{\lambda_0 z}{\varepsilon_0}$ ; $\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_0}{r}}$

for $R_1<r<R_2$ $\boxed{E=0 \text{ (inside a cond.)}}$

for $r>R_2$ $E 2\pi r z = \frac{\lambda_0 z}{\varepsilon_0}$ ; $\boxed{E=\frac{1}{2\pi\varepsilon_0}\frac{\lambda_0}{r}}$




B. What is the linear charge density $\Lambda_1$ at $R_1$ and $\Lambda_2$ at $R_2$?

$\varphi_E = \int \vec{E}\, d\vec{A}$ ; $E=0$ inside a cond. ; $\varphi_E = 0$ inside

$\varphi_E = \frac{Q_{encl}}{\varepsilon_0} = 0$ ; $Q_{encl}=0$ ; $\lambda_0 + \lambda_1 = 0$

$\boxed{\lambda_1 = -\lambda_0}$ ; $\lambda_1 + \lambda_2 = 0 \Rightarrow \boxed{\lambda_2 = \lambda_0}$

C. What is the flux of electric field through the cylindrical Gaussian surface that has radius $(R_1+R_2)/2$? Make sure to show your work and explain your reasoning.

| LO | P | F |
|---|---|---|
| 6.2 | | |
| 15.1 | | |
| 16.1 | | |
| 17.1 | | |
| 18.1 | | |
| 19.1 | | |
| 19.2 | | |
| 19.3 | | |

cyl. with $\frac{R_1+R_2}{2}$ rad. is a surface inside a conductor (plus end caps)

$E = 0$ inside a cond.

$\varphi_E = \int \vec{E}\cdot d\vec{A}$

$\boxed{\varphi_E = 0}$ because $E=0$ inside a cond. and $\vec{E} \perp \hat{n}$ at the end caps.

($\hat{n}$ is a normal unit vector to the surface)