208 Mid-Term 2 POINTS TABLE	
Multiple Choice (out of 20)	
Problem 1 (out of 20)	
Problem 2 (out of 20)	
Problem 3 (out of 20)	
Problem 4 (out of 20)	
TOTAL SCORE (out of 100)	

Multiple Choice Answers:

Version A:

1. B, D
2. A
3. A
4. A

Version B:

1. B, E
2. A
3. C
4. B

Version A:

1. **(20 marks)** Two capacitors $C_1 = 4\ \mu F$ and $C_2 = 12\ \mu F$ are connected in series across a 12-V battery. They are carefully disconnected so that they are not discharged and are reconnected with positive plate to positive plate and negative plate to negative plate.

   a) Find the potential difference across each capacitor after they are reconnected.

   b) Find the initial and final energies stored in each of the two capacitors, *i.e.* before they are disconnected and after they are reconnected.

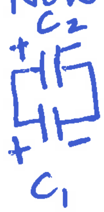
a.) 

In series, $Q_1 = Q_2 = Q$

$V_1 + V_2 = \mathcal{E} = 12\ V$

$$\Rightarrow \frac{Q}{C_1} + \frac{Q}{C_2} = 12V \Rightarrow Q = \frac{12V(C_1C_2)}{C_1+C_2} = 36\mu C$$ [Version B: $Q = 9\mu C$]

Now they are reconnected:



with $Q_{total} = Q_1 + Q_2 = 72\mu C$ [version B: $Q = 18\mu C$]

$\boxed{V_1 = V_2 = V}$ (charge redistributes btw. the two capacitors)

$\Rightarrow Q_{total} = C_1 V + C_2 V = 72\mu C$

$4\mu F \cdot V + 12\mu F \cdot V = 72\mu F \Rightarrow \boxed{V = 4.5V}$

[version B: $V = \frac{18\mu C}{8\mu F} = \boxed{2.25V}$

b.) Initially, $Q_1 = Q_2 = 36\mu C$

$U_1 = \frac{1}{2} Q_1^2/C_1 = \frac{1}{2}\frac{(36\mu C)^2}{4\mu F} = \boxed{162\ \mu J}$

$U_2 = \frac{1}{2}\frac{Q_2^2}{C_2} = \frac{1}{2}\frac{(36\mu C)^2}{12\mu F} = \boxed{54\mu J}$

Finally, $V_1 = V_2 = 4.5V$

$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(4\mu F)(4.5V)^2 = \boxed{40.5\mu J}$

$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(12\mu F)(4.5V)^2 = \boxed{121.5\mu J}$

[Version B:

$U_1 = \frac{1}{2}\frac{(9\mu C)^2}{2\mu F} = \boxed{20.25\mu J}$

$U_2 = \frac{1}{2}\frac{(9\mu C)^2}{6\mu F} = \boxed{6.75\mu J}$

$U_1 = \frac{1}{2}(2\mu F)(2.25V)^2 = \boxed{5.06\mu J}$

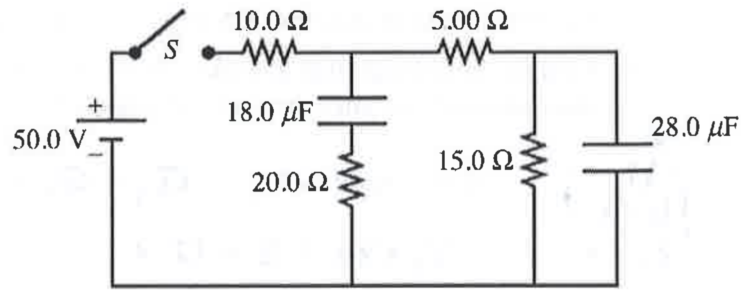
$U_2 = \frac{1}{2}(6\mu F)(2.25V)^2 = \boxed{15.2\mu J}$]

Version A:

2. (20 marks) For the circuit shown in the figure, the capacitors are all initially uncharged, the connecting leads have no resistance, the battery has no appreciable internal resistance, and the switch *S* is originally open.

a) Just after closing the switch *S*, what is the current in the 15.0-Ω resistor?

b) After the switch *S* has been closed for a very long time, what is the potential difference across the 28.0-μF capacitor?



a.) At $t=0$, $Q_c = 0$

$\Rightarrow V(\text{across } 28\,\mu F\text{-capacitor}) = 0$

$\Rightarrow V(\text{across } 15\,\Omega \text{ resistor}) = 0$

$\Rightarrow \boxed{I(\text{across } 15\text{-}\Omega \text{ resistor}) = 0}$

b.) as $t \to \infty$, $I_c \to 0$

$\Rightarrow$ All current goes through branch w/ 15-Ω resistor.

$\Rightarrow 50V - 10\Omega \cdot I - 5\Omega \cdot I - 15\Omega \cdot I = 0$

$\Rightarrow I = \frac{50V}{30\Omega} = 1.67A$

$\Rightarrow V_{15\Omega} = I(15\Omega)$
$= 1.67A(15\Omega)$
$= 25V$

$\Rightarrow \boxed{V_{28\mu F} = V_{15\Omega} = 25V}$

(version B:

$I = \frac{100V}{60\Omega} = 1.67A$

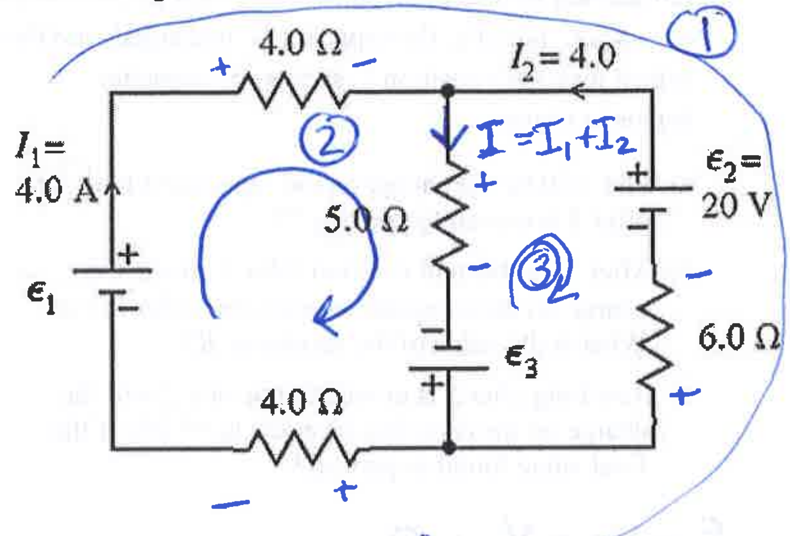
$V_{30\Omega} = 1.67A(30\Omega)$
$= 50V$

$\Rightarrow \boxed{V_{56\mu F} = V_{30\Omega} = 50V}$)

Version A:

3. (20 marks) Consider the circuit shown in the figure. Note that two currents are shown. Calculate the emfs ε_1 and ε_3.

$I = I_1 + I_2 = 8A$



Loop 1: $\mathcal{E}_1 - 4A(4\Omega) - 20V + 6\Omega(4A) - 4\Omega(4A) = 0$

$\boxed{\mathcal{E}_1 = 28V}$

Loop 2: $\mathcal{E}_1 - 4A(4\Omega) - 5\Omega \cdot I + \mathcal{E}_3 - 4A(4\Omega) = 0$

$28V - 16V - 40V + \mathcal{E}_3 - 16V = 0$

$\Rightarrow \boxed{\mathcal{E}_3 = 44V}$

OR Loop 3: $\mathcal{E}_3 - 6\Omega(4A) + 20V - 5\Omega(8A) = 0$

$\mathcal{E}_3 = 44V$

[Version B: Loop 1: $\mathcal{E}_1 - 8A(8\Omega) - 40V + 12\Omega(8A) - 8\Omega(8A) = 0$

$\boxed{\mathcal{E}_1 = 72V}$

Loop 2: $\mathcal{E}_1 - 8A(8\Omega) - 16A(10\Omega) + \mathcal{E}_3 - 8\Omega(8A) = 0$

$72V - 64V - 160V + \mathcal{E}_3 - 64V = 0$

$\boxed{\mathcal{E}_3 = 216V}$]

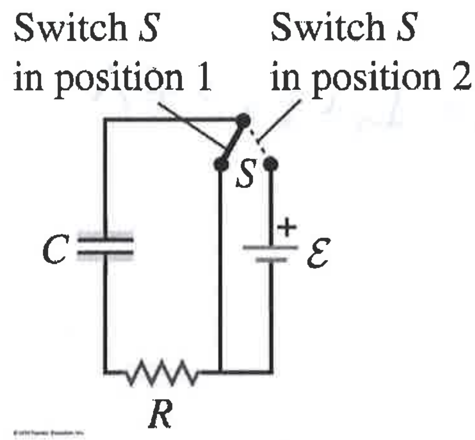
Version A:

4. **(20 marks)** In the circuit shown, $C = 11.8\ \mu F$, $\varepsilon = 56.0$ V, and the emf has negligible resistance. Initially, the capacitor is uncharged, and the switch S is in position 1. The switch is then moved to position 2, so that the capacitor begins to charge.

   a) What will be the charge on the capacitor a long time after S is moved to position 2?

   b) After S has been in position 2 for 1.50 ms, the charge on the capacitor is measured to be 110 μC. What is the value of the resistance R?

   c) How long after S is moved to position 2 will the charge on the capacitor be equal to 99.0% of the final value found in part (a)?



$\mathcal{E} - V_c - V_R = 0$

a.) as $t \to \infty$, $I \to 0$ $\Rightarrow \mathcal{E} - V_c = 0$ $\Rightarrow V_c = \mathcal{E} = 56V$

$Q_c = CV_c = 11.8\mu F(56V) = \boxed{660.8\ \mu C}$

[Version B: $Q_c = 5.90\mu F(28V) = \boxed{165.2\mu C}$]

b.) $Q(t = 1.50ms) = 110\ \mu C$

$Q(t) = Q_{t\to\infty} \cdot (1 - e^{-t/\tau})$, $\tau = RC$

$\frac{110\mu C}{660.8\mu C} = 1 - e^{-1.5ms/\tau}$ $\Rightarrow \frac{1.5ms}{\tau} = 0.182$

$\Rightarrow \tau = 8.24ms = RC = R(11.8\mu F)$

$\Rightarrow \boxed{R = 698\ \Omega}$

[Version B: $\frac{110\mu C}{165.2\mu C} = 1 - e^{-1.5ms/\tau}$

$\Rightarrow \tau = 1.37ms = R(5.90\mu F)$

$\Rightarrow \boxed{R = 232\ \Omega}$]

c.) $\frac{Q(t)}{Q_{t\to\infty}} = 0.99 = 1 - e^{-t/\tau}$ $\Rightarrow e^{-t/\tau} = 0.01 \Rightarrow e^{-t/8.24ms} = 0.01$

$\rightarrow \boxed{t = 37.9ms}$

[Version B: $e^{-t/1.37ms} = 0.01$ $\Rightarrow \boxed{t = 6.31\ ms}$]