Multiple Choice:

Version A

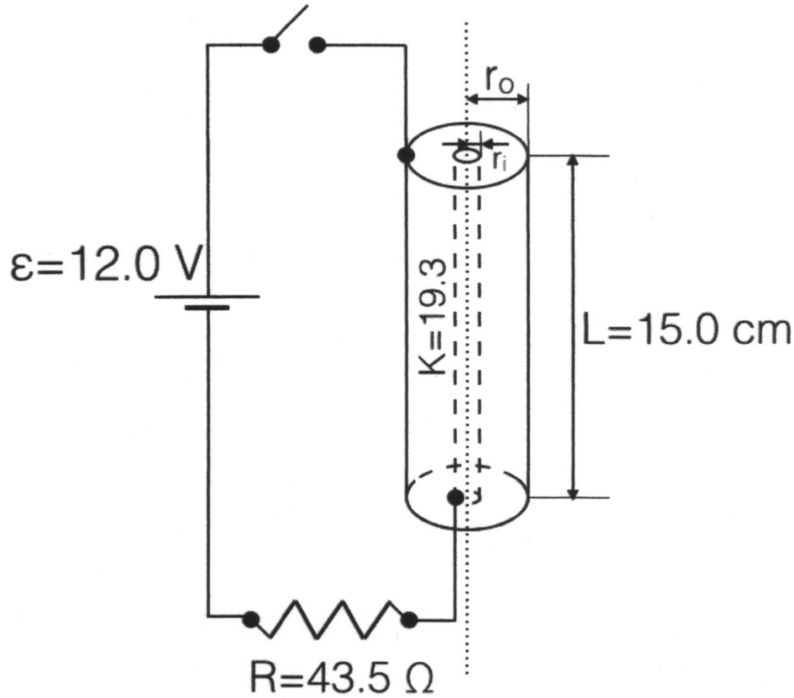
1) xii.
2) ii.
3) v.
4) i.

Version B

1) vi.
2) x.
3) vi.
4) ii.

Problem II. (20 points)

A cylindrical capacitor of length **L** = **15.0** cm with inner radius of $\mathbf{r_i}$ = **0.20** cm and outer radius of $\mathbf{r_o}$**=1.0** cm has an insulating material between the plates with a dielectric constant of **K=19.3**. It is connected to an emf **ε=12.0** V that has negligible internal resistance through a resistor **R=43.5 Ω** and a switch (as shown in the figure). The capacitor is initially uncharged and the switch is open (OFF position).



a) Find capacitance of the capacitor.

b) How much time does it take to charge this capacitor to 90% of its final charge after the switch is closed (ON position).

a) $C = \frac{Q}{V}$ - definition of a capacitance

$Q = \lambda L$ - λ is linear charge density, $\lambda = \frac{Q}{L}$

Use Gauss's law to find field due to linear charge dist.: $\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon}$; $\varepsilon = K\varepsilon_0$

inside cap.: $E\, 2\pi r l = \frac{\lambda l}{\varepsilon}$; if $r < r_o$ $E = \frac{1}{2\pi\varepsilon}\frac{\lambda}{r}$

$$V_o - V_i = -\int_{r_i}^{r_o} \vec{E}\cdot d\vec{r} = -\frac{1}{2\pi\varepsilon}\lambda\int_{r_i}^{r_o}\frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon}\ln\left(\frac{r_o}{r_i}\right)$$

$$\boxed{C = \frac{Q}{\frac{Q}{2\pi\varepsilon L}\ln\left(\frac{r_o}{r_i}\right)} = 2\pi\varepsilon L\frac{1}{\ln\left(\frac{r_o}{r_i}\right)} = 2\pi K\varepsilon_0 L\frac{1}{\ln\left(\frac{r_o}{r_i}\right)}}$$

Ver. A	Ver. B
$C = 100.\ pF = 0.10\ nF$	$C = \cancel{483}\ 227\ pF = \cancel{0.48}\ 0.23\ nF$

6) charging of a capacitor:

$$Q(t) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$

90% of the final charge $\frac{Q'}{Q_f} = 0.9$

$$\frac{Q'}{Q_f} = 1 - e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = 1 - \frac{Q'}{Q_f} \Rightarrow -\frac{t}{RC} = \ln\left(1 - \frac{Q'}{Q_f}\right)$$

$\ln$ $\ln$

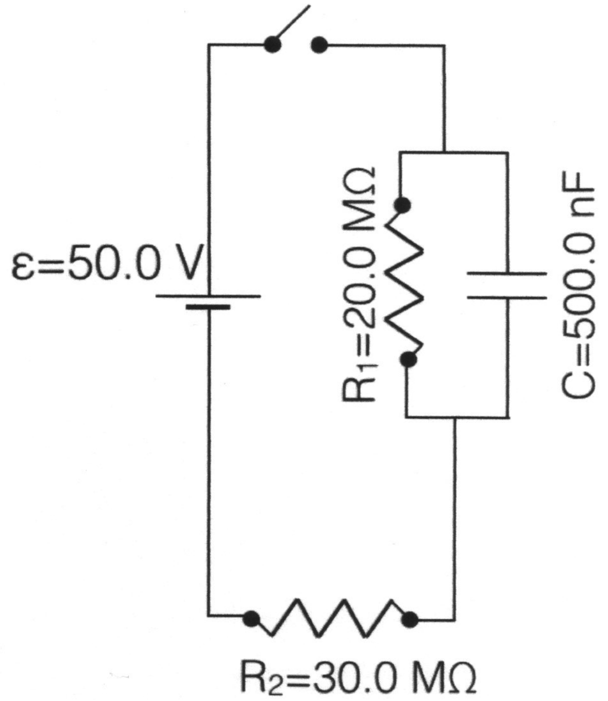
$$t = -RC\ln\left(1 - \frac{Q'}{Q_f}\right)$$

$$\boxed{t = -RC\ln(0.1)}$$

Ver. A	Ver. B
$t = 10.0$ ns	$t = \cancel{50.4}$ 49.8 ns

Problem III. (20 points)

A circuit that consists of a switch, a source of emf **ℰ=50.0** V with negligible internal resistance, resistors **R_1=20.0** MΩ and **R_2=30.0** MΩ, and a capacitor **C=500.0** nF is shown in the figure.



a) Find power dissipating in the resistor R_1 after the switch has been closed for **a very long time** (consider infinite time).

b) After the switch has been closed for **a very long time** it is opened. Find charge on the capacitor plates **5.00** seconds after the switch was opened (OFF position).

a) After the cap. is fully charged

$$\mathcal{E} = I(R_1+R_2) \; ; \; I = \frac{\mathcal{E}}{R_1+R_2}$$

$$P_{R_1} = I^2 R_1 = \frac{\mathcal{E}^2}{(R_1+R_2)^2} R_1$$

Ver. A $\boxed{P_{R_1} = \frac{50.0^2}{(20.0+30.0)^2\, 10^{+6}} \cdot 20.0 = 200\mu W}$

ver. B $\boxed{P_{R_1} = \frac{100.0^2}{(80.0+20.0)^2\, 10^6} \cdot 80.0 = 80.0\ \mu W}$

b) Find charge on the cap. before the switch is opened: $Q_i = C \cdot V$; $V = I R_1 = \frac{\mathcal{E}}{R_1+R_2} \cdot R_1$

$$Q_i = \frac{\mathcal{E} \cdot C \cdot R_1}{R_1+R_2}$$

discharge through R_1 $\quad Q(t) = Q_i \cdot e^{-\frac{t}{R_1C}}$

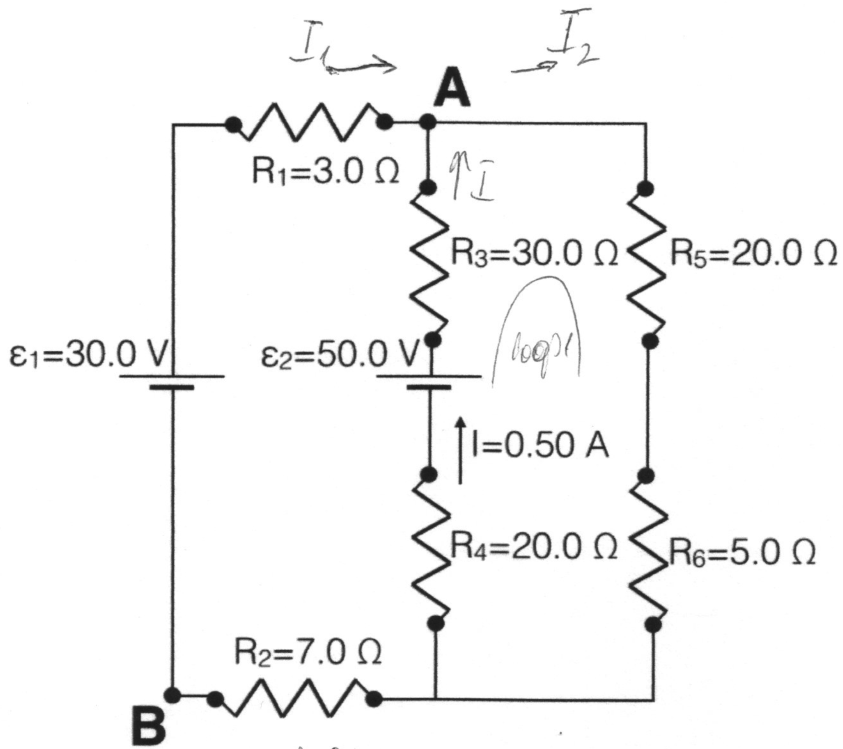
$$\boxed{Q(t) = \frac{\mathcal{E} C R_1}{R_1+R_2} e^{-\frac{t}{R_1C}}}$$

Ver. A $\boxed{Q(t=5s) = 6.07\ \mu C}$

Ver. B $\boxed{Q(t=4s) = 3.33\ \mu C}$

Problem IV. (20 points)

The circuit is shown in the figure below. All emfs have negligible internal resistances. Current through the second emf $\mathbf{\varepsilon_2}$ is 0.50 A. Find potential difference between points **A** and **B** (V_A - V_B).



Apply Kirchhoff's rules:

junction A: $I_2 = I_1 + I$

loop 1: $\mathcal{E}_2 - IR_3 - I_2R_5 - I_2R_6 - IR_4 = 0$

$$I_2 = \frac{1}{R_5+R_6}\left(\mathcal{E}_2 - I(R_3+R_4)\right)$$

$$I_2 = \frac{1}{25.0}\left(50.0 - 0.50(50.0)\right) = 1.00\ A$$

$$I_1 = I_2 - I = 0.5\ A$$

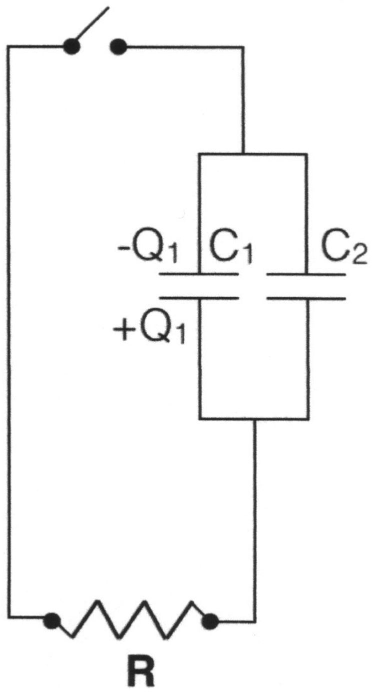
Ver. A $\quad V_A - V_B = \mathcal{E}_1 - I_1R_1 = 30.0 - 0.5 \cdot 3.0 = +28.5\ V$

ver. B $\quad V_A - V_B = \mathcal{E}_1 - I_1R_2 = 30.0 - 0.5 \cdot 7.0 = +26.5\ V$

Ver. A

Problem V. (20 points)

A switch, two capacitors (capacitances C_1 and C_2 are known) and a resistor **R** are connected as shown in the figure. The capacitors are charged and it is known that the C_1 has charge Q_1 on the plates when the switch is open. Find **total** energy dissipated in the resistor **R** after the switch is closed (assume that the switch remains closed for a very long time, sufficient for complete discharge of the capacitors).



Total energy stored

$U_{tot} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

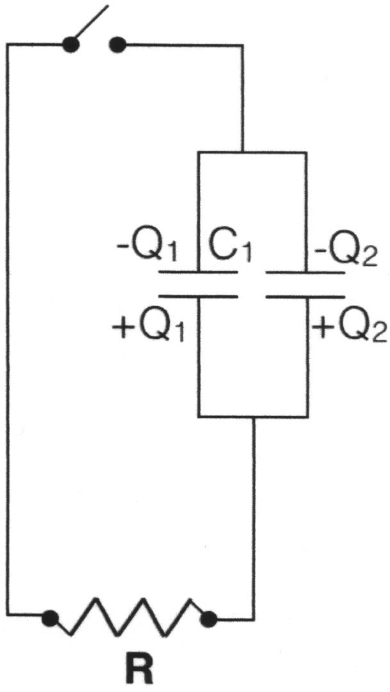
$V = \frac{Q_1}{C_1}$ - when the switch is open

$U_{tot} = \frac{1}{2}(C_1+C_2)\cdot V = \frac{1}{2}(C_1+C_2)\frac{Q_1^2}{C_1^2}$

This energy will be dissipated in the resistor R when the switch is closed

$$\boxed{E_{dis} = \frac{1}{2}(C_1+C_2)\frac{Q_1^2}{C_1^2}}$$

Ver. B



Problem V. (20 points)

A switch, two capacitors and a resistor **R** are connected as shown in the figure. Capacitance $\mathbf{C_1}$ of the first capacitor is known but capacitance of the second one is **NOT** known. However, it is known that charge on the plates of the two capacitors is $\mathbf{Q_1}$ and $\mathbf{Q_2}$ when the switch is open. Find **total** energy dissipated in the resistor **R** after the switch is closed (assume that the switch remains closed for a very long time, sufficient for complete discharge of the capacitors).

Total energy stored

$$U_{tot} = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2};$$

$$V = \frac{Q_1}{C_1}; \quad V = \frac{Q_2}{C_2}; \quad C_2 = \frac{Q_2}{V} = \frac{Q_2}{Q_1}C_1$$

$$U_{tot} = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^{\cancel{2}}}{\cancel{Q_2}}\frac{Q_1}{C_1} = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2 Q_1}{C_1}$$

This energy will be dissipated in the resistor R when the switch is closed

$$\boxed{E_{dis} = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2 Q_1}{C_1} = \frac{1}{2}\frac{Q_1}{C_1}(Q_1+Q_2)}$$