Physics 208, Spring 2014 - Exam #2

Name (Last, First): ______________________

ID#: ______________________

Section #: ______________________

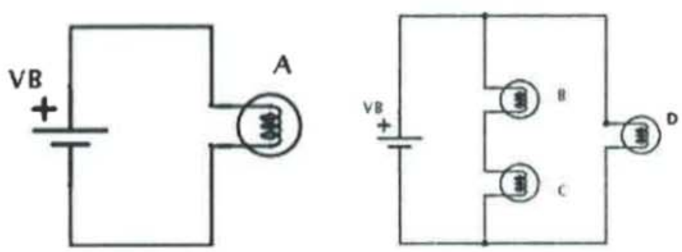
Key Ver. A

- You have 75 minutes to complete the exam
- Formulae are provided on a separate colored sheet. You may not use any other formula sheet.
- You may use only a simple calculator: one without memory, or with a memory demonstrated to be cleared.
- When calculating numerical values, be sure to keep track of units. Results must include proper units.
- Be alert to the number of significant figures in the information given. Results must have the correct number of significant figures.
- If you are unable to solve a problem whose solution is needed in another problem, then assign a symbol for the solution of the first problem and use that symbol in solving the second problem.
- If you need additional space to answer a problem, use the back of the sheet it is written on. Make sure to write a note for the grader that the solution is continued on the other side.
- Show your work. Without supporting work, the answer alone is worth nothing.
- Mark your answers clearly by drawing boxes around them.
- Please write clearly. You may gain marks for a partially correct calculations if your work can be deciphered.

Key Ver. A

Problem I. MULTIPLE CHOICE (The next four questions) (20 points)

1. In the two circuits given below, all light bulbs are identical. The batteries are also identical. Which of the following correctly indicate the relative brightness of the bulbs (A > B implies bulb A is brighter than bulb B). (5 points)



A) A > B>C>D

B) A>(B=C)>D

C) D>(B=C)>A

D) D>A>(B=C)

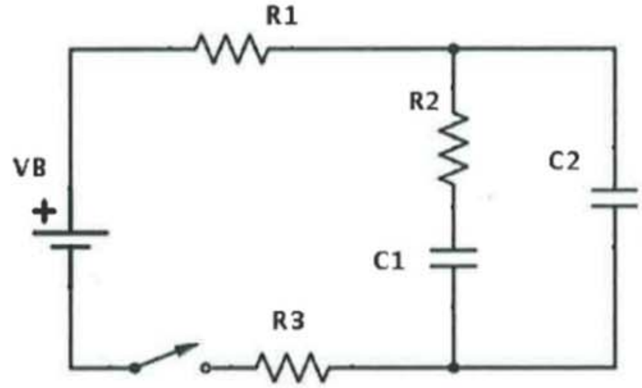
E) A>D>(B=C)

F) (B=C)>D>A

G) (A=D)>(B=C)

H) A=B=C=D since all bulbs are identical and therefore have the same power rating.

2. In the circuit given below, *both* capacitors are initially *uncharged* and V_B=15V, R_1=3Ω, R_2=5Ω, R_3=7Ω, C_1=5μF and C_2=8μF. How much current will flow through R_1 *just after* the switch is closed? (5 points)



A) 0.00 A

B) 0.75A

C) 1.00A

D) 1.50A

E) 5.00A

3. A 120V mains electric outlet has a 15Amp circuit breaker (fuse). A 1000 W refrigerator is plugged into this outlet. You want to plug in a few more devices on to the same mains outlet. You have 3 devices with power ratings of 700W, 850W, 950W. Which of these can you plug in without blowing the fuse. (5 points)

A) The 700W device only.

B) Either the 700W or the 850W but not both.

C) Both the 700W and the 850W devices can be plugged in together.

D) Either one of the 700W, 850W or 950W device but not any two of them.

E) None of these. We need a device less than 700W.

4. A capacitor with no dielectric between its plates is connected in series with a resistor and a battery and allowed to acquire the maximum possible charge for this set up. With the capacitor fully charged and the battery still connected to the circuit, a dielectric is inserted between the plates of the capacitor (assume instantaneously) and the circuit remains connected in this set up for a very long time thereafter (assume infinite time).

If Q_c is the charge on the capacitor and V_c the potential drop across its plates, then which of the following statement is true for Q_c and V_c after the dielectric is inserted as compared to the case without the dielectric? (5 points)

A. Q_c increases but V_c decreases.

B. Q_c decreases but V_c increases.

C. Q_c increases but V_c remains the same.

D. Q_c decreases but V_c remains the same.

E. Q_c remains the same but V_c increases.

F. Q_c remains the same but V_c decreases.

G. Both Q_c and V_c increases.

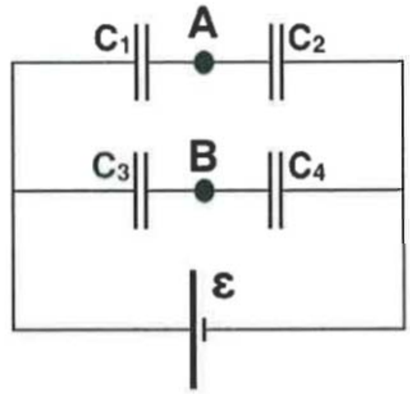
H. Both Q_c and V_c decreases.

Problem II. (20 points)

A DC-circuit that has a source of emf and four different capacitors is shown in the figure below. Assume that the capacitors are fully charged and that there is no current in the circuit. $\mathcal{E}$=200.0 V, C_1=10.0 μF, C_2=20.0 μF, C_3=30.0 μF and C_4=50.0 μF.

a) Find charge on each capacitor.

b) Find potential difference V_{AB} between points A and B.



a) C_1 and C_2 are in series:

$Q_1 = Q_2 = \mathcal{E}\cdot C_{eff_{12}}$;

$\frac{1}{C_{eff_{12}}} = \frac{1}{C_1} + \frac{1}{C_2}$; $C_{eff_{12}} = \frac{C_1 C_2}{C_1 + C_2}$

$\boxed{Q_1 = Q_2 = \mathcal{E}\frac{C_1 C_2}{C_1 + C_2}}$; $\boxed{Q_1 = Q_2 = 200.0[V]\cdot\frac{10\cdot 20}{10+20}\cdot 10^{-6}[F] = 1.33\cdot 10}$

$Q_3 = Q_4 = \mathcal{E}\cdot C_{eff_{34}}$ (C_3 and C_4 are in series)

$\frac{1}{C_{eff_{34}}} = \frac{1}{C_3} + \frac{1}{C_4}$; $C_{eff_{34}} = \frac{C_3 C_4}{C_3 + C_4}$

$\boxed{Q_3 = Q_4 = \mathcal{E}\frac{C_3 C_4}{C_3 + C_4}}$; $Q_3 = Q_4 = 200.0[V]\cdot\frac{30\cdot 50}{30+50}\cdot 10^{-6}[F]$

$\boxed{Q_3 = Q_4 = 3.75\cdot 10^{-3} C = 3.75 mC}$

b) $V_{AB} = V_A - V_B$; $V_A = \mathcal{E} - V_1$; $V_1 = \frac{Q_1}{C_1}$; $Q_1 = \mathcal{E}\frac{C_1 C_2}{C_1 + C_2}$

$V_B = \mathcal{E} - V_3$; $V_3 = \frac{Q_3}{C_3}$; $Q_3 = \mathcal{E}\frac{C_3 C_4}{C_3+C_}$

$$V_{AB} = \left(\mathcal{E} - \mathcal{E}\frac{C_2}{C_1 + C_2}\right) - \left(\mathcal{E} - \mathcal{E}\frac{C_4}{C_3 + C_4}\right) = \mathcal{E}\left(\frac{C_4}{C_3 + C_4} - \frac{C_2}{C_1 + C_2}\right)$$

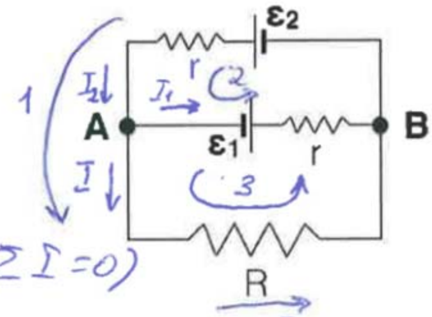
$$V_{AB} = \mathcal{E}\frac{C_4(C_1 + C_2) - C_2(C_3 + C_4)}{(C_3 + C_4)(C_1 + C_2)} = \boxed{\mathcal{E}\frac{C_4 C_1 - C_2 C_3}{(C_1 + C_2)(C_3 + C_4)}}$$

$$\boxed{V_{AB} = 200.0[V]\cdot\frac{50\cdot 10 - 20\cdot 30}{(10+20)(30+50)} = -8.33 V}$$

Problem III. (20 points)

A DC circuit that consist of two sources of emf with internal resistance **r** (each) and a load resistor R is shown in the figure below. Emf of the sources, their internal resistances and resistance of the load are known: ε_1=15.0 V, **r**=5.0 Ω, ε_2=50.0 V and **R**=15.0 Ω.

a) Find magnitude and direction of current through the load resistor R.
b) Find power dissipated on load resistor.
c) Find potential difference V_{AB} between points A and B



a) Apply kirchhoff's rules:

Junction A: $I_2 - I - I_1 = 0$ $(\Sigma I = 0)$
loop 1: $\varepsilon_2 - I_2 r - IR = 0$
loop 2: $\varepsilon_2 - I_2 r - I_1 r + \varepsilon_1 = 0$

$\varepsilon_2 + \varepsilon_1 - I_2 r - (I_2 - I)\cdot r = 0$ (from loop2 and Jun. A)

$\varepsilon_2 + \varepsilon_1 - 2I_2 r + Ir = 0$

from loop 1: $I_2 r = \varepsilon_2 - IR$

$\varepsilon_2 + \varepsilon_1 - 2(\varepsilon_2 - IR) + Ir = 0$

$\varepsilon_2 + \varepsilon_1 - 2\varepsilon_2 + 2IR + Ir = 0$; $I = \dfrac{\varepsilon_2 - \varepsilon_1}{2R + r}$

$I = \dfrac{50.0 - 15.0\ [V]}{2\cdot 15 + 5\ [\Omega]} = 1.0A$

direction is as shown in the fig. from left to right.

b) $P = I^2 R$; $P = \left(\dfrac{\varepsilon_2 - \varepsilon_1}{2R + r}\right)^2 \cdot R$

$P = (1.0\ [A])^2 \cdot 15\ [\Omega] = 15.0W$

c) $V_{AB} = V_A - V_B = IR = \dfrac{\varepsilon_2 - \varepsilon_1}{2R + r} \cdot R$

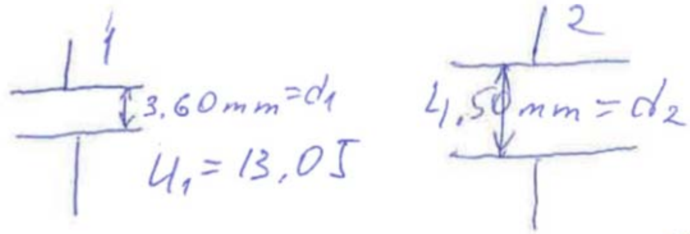
$V_{AB} = (1.0\ [A]) \cdot 15\ [\Omega] = 15.0V$

Problem IV. (20 points)

A parallel-plate vacuum capacitor has 13.0 J of energy stored in it. The separation between the plates is 3.60 mm. If the separation is increased to 4.50 mm, what is the energy stored

a) if the capacitor is disconnected from the potential source before moving the plates
b) if the capacitor remains connected to the potential source during and after the plates are moved.

a)



$U_2 - ?$

$Q_1 = Q_2 = const = Q$

$C = \varepsilon_0 \frac{A}{d}$; $U = \frac{Q^2}{2C}$

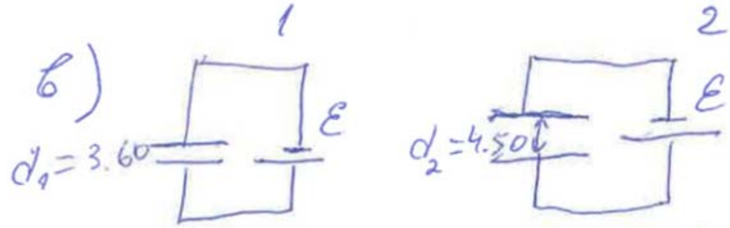
$U_1 = \frac{Q^2}{2C_1}$; $C_1 = \varepsilon_0 \frac{A}{d_1}$

$U_2 = \frac{Q^2}{2C_2}$; $C_2 = \varepsilon_0 \frac{A}{d_2}$

$\frac{U_1}{U_2} = \frac{C_2}{C_1}$; $\frac{U_1}{U_2} = \frac{\varepsilon_0 A}{d_2} \frac{d_1}{\varepsilon_0 A} = \frac{d_1}{d_2}$

$U_2 = U_1 \frac{d_2}{d_1}$; $U_2 = 13.0[J] \cdot \frac{4.50}{3.60} = 16.3 J$

b)



$V_1 = V_2 = const = V$

$U = \frac{Q^2}{2C}$; $Q = CV$; $U = \frac{1}{2}V^2C$; $\frac{U_1}{U_2} = \frac{C_1}{C_2} = \frac{d_2}{d_1}$

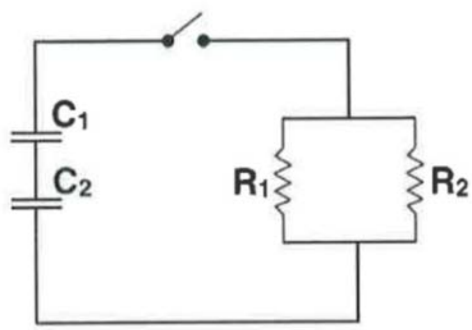
$U_2 = U_1 \frac{d_1}{d_2} = 13[J] \cdot \frac{3.60}{4.50} = 10.4[J]$

Problem V. (20 points)

In the circuit shown in the Figure below both capacitors are initially charged so that the charge on the positive plate of each capacitor is Q_i=0.45 mC. $\mathbf{C_1}$=20.0 μF, $\mathbf{C_2}$=30.0 μF, $\mathbf{R_1}$=30.0 Ω and $\mathbf{R_2}$=50.0 Ω.

a) How long after closing the switch will it take for the charge on the positive plates of the capacitors to be reduced to 0.10 mC?

b) What will be the current through the switch at that time?



a) For discharge:

$$Q(t) = Q_i e^{-t/\tau}$$

$$\tau = R_{eff} C_{eff}$$

capacitors are in series:

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}; \quad C_{eff} = \frac{C_1 C_2}{C_1 + C_2}$$

resistors are in parallel:

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}; \quad R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\boxed{\tau = \frac{R_1 R_2 C_1 C_2}{(R_1+R_2)(C_1+C_2)}} \qquad \boxed{\tau = 2.25 \cdot 10^{-4}\,s}$$

$$Q' = Q_i e^{-t/\tau}; \quad e^{-\frac{t}{\tau}} = \frac{Q'}{Q_i}; \quad -\frac{t}{\tau} = \ln \frac{Q'}{Q_i}$$

$$\boxed{t = -\tau \ln \frac{Q'}{Q_i} = -\frac{R_1 R_2 C_1 C_2}{(R_1+R_2)(C_1+C_2)} \ln \frac{Q'}{Q_i}}$$

$$\boxed{t = -2.25 \cdot 10^{-4} \cdot \ln \frac{0.1}{0.45} = 3.38 \cdot 10^{-4}\,s}$$

b) $$i(t) = \frac{dq}{dt}; \quad \boxed{i(t) = \frac{Q_i}{\tau} e^{-\frac{t}{\tau}}}$$

$$i(t = 3.38 \cdot 10^{-4}\,s) = \frac{0.45 \cdot 10^{-3}\,C}{2.25 \cdot 10^{-4}\,s} e^{-\frac{3.38 \cdot 10^{-4}}{2.25 \cdot 10^{-4}}}$$

$$\boxed{i(t = 3.38 \cdot 10^{-4}) = 4.45 \cdot 10^{-1}\,A}$$