208 Mid-Term 1 POINTS TABLE	
Multiple Choice (out of 20)	
Problem 1 (out of 20)	
Problem 2 (out of 20)	
Problem 3 (out of 20)	
Problem 4 (out of 20)	
TOTAL SCORE (out of 100)	

Multiple Choice Answers:

Version A:

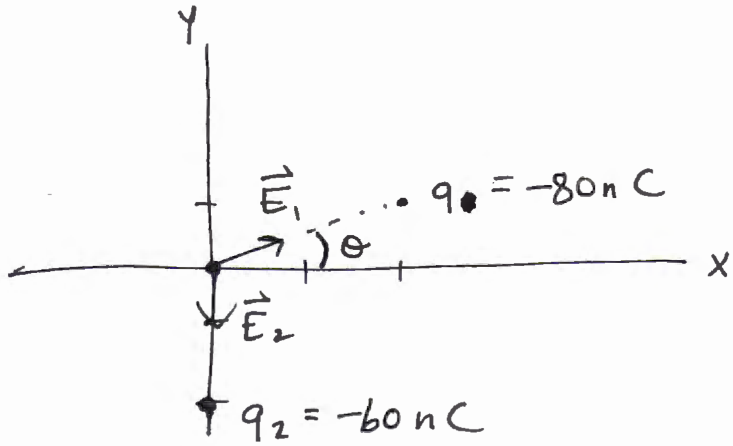
MC1. D
MC2. B
MC3. D, E
MC4. C

Version B:

MC1. C
MC2. D
MC3. B, C
MC4. B

Version A

1. **(20 marks)** A point charge $q_1 = -80.0$ nC is located at $(x,y) = (2.00$ m, 1.00 m$)$, and a point charge $q_2 = -60.0$ nC is located at $(x,y) = (0.00, -2.00$ m$)$. Determine the net electric field, magnitude and direction, at the origin of the (x,y) coordinate system.

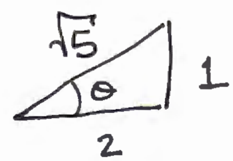


$$E_{1,x} = +\frac{k|q_1|}{r^2}\cos\theta$$

$$E_{1,y} = +\frac{k|q_1|}{r^2}\sin\theta$$

$$E_{2,x} = 0$$

$$E_{2,y} = -\frac{k|q_2|}{r^2}$$



$$\cos\theta = \frac{2}{\sqrt{5}}$$

$$\sin\theta = \frac{1}{\sqrt{5}}$$

$$E_{1,x} = +\frac{(9\times10^9)(80\,nC)}{(\sqrt{5}\,m)^2}\cdot\frac{2}{\sqrt{5}}$$

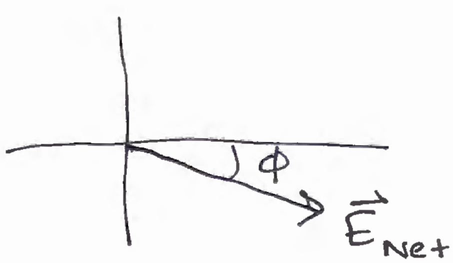
$$= 129\ N/C$$

$$E_{1,y} = \frac{(9\times10^9)(80\,nC)}{(\sqrt{5}\,m)^2}\frac{1}{\sqrt{5}} = 64.4\ N/C$$

$$E_{2,y} = -\frac{(9\times10^9)(60\,nC)}{(2m)^2} = -135\ N/C$$

$$E_{Net,x} = E_{1,x} + E_{2,x} = 129\ N/C$$

$$E_{Net,y} = E_{1,y} + E_{2,y} = 64.4\ N/C - 135\ N/C = -70.6\ N/C$$



$$|\vec{E}_{Net}| = \sqrt{(129\ N/C)^2 + (70.6\ N/C)^2}$$

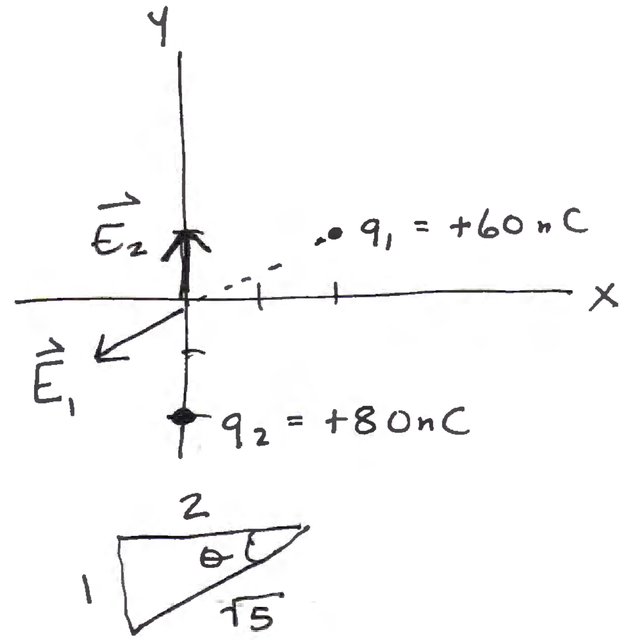
$$= 147\ N/C$$

$$\phi = \tan^{-1}\left(\frac{70.6}{129}\right) = 28.7^\circ$$

below the horizontal

Version B

1. **(20 marks)** A point charge $q_1 = +60.0$ nC is located at $(x,y) = (2.00$ m, 1.00 m$)$, and a point charge $q_2 = +80.0$ nC is located at $(x,y) = (0.00, -2.00$ m$)$. Determine the net electric field, magnitude and direction, at the origin of the (x,y) coordinate system.



$$E_{1,x} = \frac{-k|q_1|}{r^2}\cos\theta$$

$$E_{1,y} = \frac{-k|q_1|}{r^2}\sin\theta$$

$$E_{2,x} = 0$$

$$E_{2,y} = +\frac{k|q_2|}{r^2}$$

$$E_{1,x} = \frac{-(9\times10^9)(60\text{nC})}{(\sqrt{5}\text{ m})^2}\cdot\frac{2}{\sqrt{5}}$$

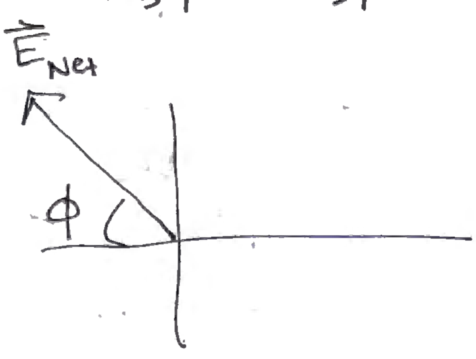
$$= -96.6 \text{ N/C}$$

$$E_{1,y} = \frac{-(9\times10^9)(60\text{nC})}{(\sqrt{5}\text{m})^2}\cdot\frac{1}{\sqrt{5}} = -48.3 \text{ N/C}$$

$$E_{2,y} = +\frac{(9\times10^9)(80\text{nC})}{(2\text{m})^2} = 180 \text{ N/C}$$

$$E_{Net,x} = E_{1,x} + E_{2,x} = -96.6 \text{ N/C}$$

$$E_{Net,y} = E_{1,y} + E_{2,y} = -48.3 \text{ N/C} + 180 \text{ N/C} = 132 \text{ N/C}$$



$$|\vec{E}_{Net}| = \sqrt{(96.6)^2 + (132)^2}$$

$$= 163 \text{ N/C}$$

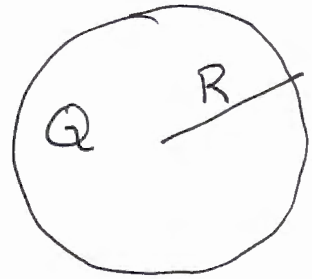
$$\phi = \tan^{-1}\left(\frac{132}{96.6}\right) \approx 53.8°$$

above the −x axis

Version A

2. (20 marks) Charge +Q is distributed evenly throughout the volume of an insulating sphere that has radius R. Assume that the potential V=0, at r=∞. Calculate the potential at a radius of r=R/3.

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$



for $r > R$, $\quad E(4\pi r^2) = \dfrac{Q}{\varepsilon_0}$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\cancel{V(r=\infty)}^{\,0} - V(r=R) = \int_{\infty}^{R} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = -\frac{Q}{4\pi\varepsilon_0 r}\Big|_{\infty}^{R}$$

$$\Rightarrow -V(r=R) = -\frac{Q}{4\pi\varepsilon_0 R} \Rightarrow V(r=R) = +\frac{Q}{4\pi\varepsilon_0 R}$$

for $r < R$, $\quad E(4\pi r^2) = \dfrac{Q_{enc}}{\varepsilon_0} = \rho\dfrac{V_{enc}}{\varepsilon_0}$

$$= \frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

$$V(r=R) - V(r=R/3) = \int_{R}^{R/3} \frac{Qr}{4\pi\varepsilon_0 R^3}\,dr = \frac{Qr^2}{8\pi\varepsilon_0 R^3}\Big|_{R}^{R/3}$$

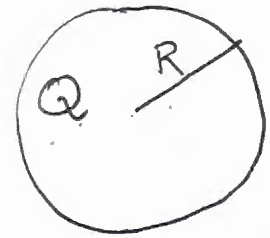
$$\Rightarrow \frac{Q}{4\pi\varepsilon_0 R} - V(r=R/3) = \frac{Q}{72\pi\varepsilon_0 R} - \frac{Q}{8\pi\varepsilon_0 R} = -\frac{Q}{9\pi\varepsilon_0 R}$$

$$\Rightarrow V(r=R/3) = \frac{Q}{4\pi\varepsilon_0 R} + \frac{Q}{9\pi\varepsilon_0 R} = \frac{13Q}{36\pi\varepsilon_0 R}$$

Version B

2. (20 marks) Charge +Q is distributed evenly throughout the volume of an insulating sphere that has radius R. Assume that the potential V=0, at r=∞. Calculate the potential at a radius of r=R/2.

$$\rho = \frac{Q}{V} = \frac{Q}{4/3\,\pi R^3}$$



for $r > R$, $E(4\pi r^2) = \dfrac{Q}{\varepsilon_0}$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\cancel{V(r=\infty)}^{\,0} - V(r=R) = \int_\infty^R \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = -\frac{Q}{4\pi\varepsilon_0 r}\Bigg|_\infty^R$$

$$\Rightarrow -V(r=R) = \frac{-Q}{4\pi\varepsilon_0 R} \Rightarrow V(r=R) = +\frac{Q}{4\pi\varepsilon_0 R}$$

for $r < R$, $E(4\pi r^2) = \dfrac{Q_{enc}}{\varepsilon_0} = \rho\dfrac{V_{enc}}{\varepsilon_0}$

$$= \frac{Q}{4/3\,\pi R^3}\cdot\frac{4/3\,\pi r^3}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

$$V(r=R) - V(r=R/2) = \int_R^{R/2} \frac{Qr}{4\pi\varepsilon_0 R^3}\,dr = \frac{Qr^2}{8\pi\varepsilon_0 R^3}\Bigg|_R^{R/2}$$

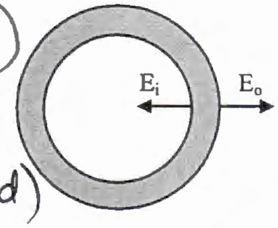
$$\frac{Q}{4\pi\varepsilon_0 R} - V(r=R/2) = \frac{Q}{32\pi\varepsilon_0 R} - \frac{Q}{8\pi\varepsilon_0 R} = \frac{-3Q}{32\pi\varepsilon_0 R}$$

$$\Rightarrow V(r=R/2) = \frac{Q}{4\pi\varepsilon_0 R} + \frac{3Q}{32\pi\varepsilon_0 R} = \frac{11Q}{32\pi\varepsilon_0 R}$$

Version

A

3. (20 marks) A hollow (but not necessarily empty) conducting spherical shell has an inner radius of 8.00 cm and an outer radius of 10.0 cm. There is an electric field at the inner surface of the shell, E_i, which has a magnitude of 80.0 N/C and points toward the center of the sphere at all points on the inner surface. The electric field at the outer surface of the shell, E_o, has a magnitude of 80.0 N/C and points away from the center of the sphere (as shown) at all points on the outer surface. Determine the magnitude of the charge on: a) the inner surface and b) the outer surface of the spherical shell.



$r_i = 0.08\ m$ $\qquad E(r_i) = -80\ \frac{N}{C}$ (inward)

$r_o = 0.10\ m$ $\qquad E(r_o) = +80\ \frac{N}{C}$ (outward)

for $r < r_i$, $\quad E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$ $\qquad$ where $Q_{enc} \equiv$ charge within cavity of spherical shell

$$\Rightarrow E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}$$

at $r \to r_i$ $\qquad E = \frac{Q_{enc}}{4\pi\varepsilon_0 r_i^2} = -80\ \frac{N}{C}$

$$\Rightarrow Q_{enc} = -80\ \frac{N}{C}(4\pi\varepsilon_0)(0.08m)^2$$
$$= -5.68 \times 10^{-11}\ C$$

for $r_i < r < r_o$, $\quad E = 0$ (inside the conductor)

$$\Rightarrow E(4\pi r^2) = \frac{Q_{enc} + Q_i}{\varepsilon_0} = 0$$

$$\Rightarrow Q_i = -Q_{enc} = \boxed{+5.68 \times 10^{-11}\ C}$$

for $r > r_o$, $\quad E(4\pi r^2) = \frac{Q_{enc} + Q_i + Q_o}{\varepsilon_0} = \frac{Q_o}{\varepsilon_0}$

$$\Rightarrow E = \frac{Q_o}{4\pi\varepsilon_0 r^2}$$

at $r = r_o$ $\qquad E = \frac{Q_o}{4\pi\varepsilon_0 r_o^2} = +80\ \frac{N}{C}$

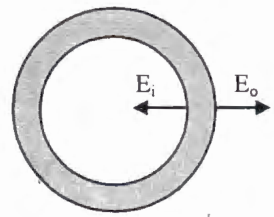
$$\Rightarrow Q_o = 80\ \frac{N}{C}(4\pi\varepsilon_0)(0.10m)^2 = \boxed{+8.89 \times 10^{-11}\ C}$$

Version B

3. (20 marks) A hollow conducting spherical shell has an inner radius of 8.00 cm and an outer radius of 10.0 cm. The electric field at the inner surface of the shell, E_i, has a magnitude of 40.0 N/C and points toward the center of the sphere, and the electric field at the outer surface, E_o, has a magnitude of 40.0 N/C and points away from the center of the sphere (as shown). Determine the magnitude of the charge on: a) the inner surface and b) the outer surface of the spherical shell.

$r_i = 0.08\,m$

$r_o = 0.10\,m$



for $r < r_i$, $E(4\pi r^2) = \dfrac{Q_{enc}}{\varepsilon_0}$ where $Q_{enc} \equiv$ charge within cavity of spherical shell

$$\Rightarrow E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}$$

at $r \to r_i$ $E = \dfrac{Q_{enc}}{4\pi\varepsilon_0 r_i^2} = -40\frac{N}{C}$ (negative for inward direction)

$$\Rightarrow Q_{enc} = -40\tfrac{N}{C}(4\pi\varepsilon_0)(0.08\,m)^2$$
$$= -2.84 \times 10^{-11}\ C$$

for $r_i < r < r_o$, $E = 0$ (inside the conductor)

$$\Rightarrow E(4\pi r^2) = \frac{Q_{enc} + Q_i}{\varepsilon_0} = 0$$

$$\Rightarrow Q_i = -Q_{enc} = \boxed{+2.84 \times 10^{-11}\ C}$$

for $r > r_o$, $E(4\pi r^2) = \dfrac{Q_{enc} + Q_i + Q_o}{\varepsilon_0} = \dfrac{Q_o}{\varepsilon_0}$

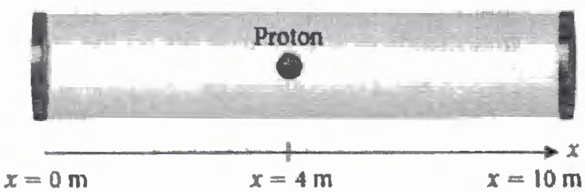
$$\Rightarrow E = \frac{Q_o}{4\pi\varepsilon_0 r^2}$$

at $r = r_o$ $E = \dfrac{Q_o}{4\pi\varepsilon_0 r_o^2} = +40\,N/C$

$$\Rightarrow Q_o = 40\,N/C\,(4\pi\varepsilon_0)(0.10\,m)^2 = \boxed{+4.44 \times 10^{-11}\ C}$$

version A

4. **(20 marks)** The electric potential inside a 10-m long linear particle accelerator is given by $V = (3000 - 5x^2/\mathrm{m}^2)$ V, where x is the distance from the left plate along the accelerator tube, as shown in the figure.



a) Determine an expression for the electric field along the accelerator tube.
b) A proton is released (from rest) at $x = 4$ m. Calculate the acceleration of the proton just after it is released.
c) What is the impact speed of the proton when (and if) it collides with the plate.

a.) $E_x = -\frac{\partial V}{\partial x} = (+10x)\frac{V}{m^2}$

$\vec{E} = (10x)\frac{V}{m^2}\,\hat{i}$

b.) $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} = \frac{q(10x)\frac{V}{m^2}}{m}\,\hat{i}$

$= \frac{1.6\times10^{-19}\,C}{1.67\times10^{-27}\,kg}\cdot 10\cdot 4m\,\frac{V}{m^2}\,\hat{i}$

$= 3.83\times10^{9}\,m/s^2\,\hat{i}$

(accelerating to the right plate)

c.) $v_o = 0$ $v_f = ?$

$W = -\Delta U = \Delta K = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_o^2}$

$U_i - U_f = \frac{1}{2}mv_f^2$

$U(x) = qV(x) = q(3000 - 5x^2/m^2)V$

$U(x=4m) - U(x=10m) = q[-5(16)V + 5(100)V]$

$= 1.6\times10^{-19}\,C\,(420V) = 6.72\times10^{-17}\,J$

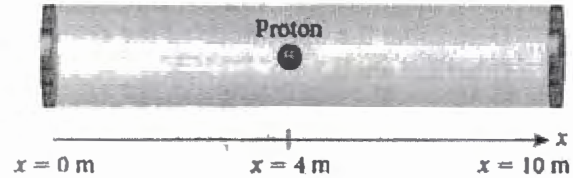
$\frac{1}{2}mv_f^2 = 6.72\times10^{-17}\,J$

$\Rightarrow v_f = \sqrt{\frac{2(6.72\times10^{-17}\,J)}{1.67\times10^{-27}\,kg}} = 2.84\times10^{5}\,\frac{m}{s}$

version B

4. **(20 marks)** The electric potential inside a 10.0-m long linear particle accelerator is given by $V = (2000 + 5x^2/\text{m}^2)$ V, where x is the distance from the left plate along the accelerator tube, as shown in the figure.

a) Determine an expression for the electric field along the accelerator tube.
b) A proton is released (from rest) at $x = 4.00$ m. Calculate the acceleration (magnitude and direction) of the proton just after it is released.
c) What is the impact speed of the proton when (and if) it collides with one of the end-plates.



a.) $E_x = -\frac{\partial V}{\partial x} = (-10x)\frac{V}{m^2}$

$\vec{E} = (-10x)\frac{V}{m^2}\,\hat{\imath}$

b.) $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} = \frac{q}{m}(-10x)\frac{V}{m^2}\,\hat{\imath}$

$= \frac{1.6\times10^{-19}C}{1.67\times10^{-27}kg}(-10\cdot 4m)\frac{V}{m^2}\,\hat{\imath}$

$= -3.83\times10^{9}\ m/s^2\,\hat{\imath}$ (accelerating to the left plate)

c.) $v_0 = 0 \qquad v_f = ?$

$W = -\Delta U = \Delta K = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_0^2}^{\,0}$

$U_i - U_f = \frac{1}{2}mv_f^2$

$U(x) = qV(x) = q(2000 + 5x^2/m^2)\,V$

$U(x=4m) - U(x=0m) = q[5(16)V - 5(0)V]$

$= 1.6\times10^{-19}C\,(80V) = 1.28\times10^{-17}\ J$

$\Rightarrow \frac{1}{2}mv_f^2 = 1.28\times10^{-17}\ J$

$v_f = \sqrt{\frac{2(1.28\times10^{-17}J)}{1.67\times10^{-27}kg}} = 1.24\times10^{5}\ m/s$