Multiple choice questions:

Question	Exam A	Exam B
MC1	vi	vi
MC2	viii	ii
MC3	a	a
MC4	g	g

1. **(20 marks)** Two 1.0-cm-diameter non-conducting spheres have a total charge of 75.0 μC (shared between them) and are placed 1.05 m apart. The spheres are not connected to each other

10 a) If the force each exerts on the other is 11.0 N and is attractive, what is the charge on each?

10 b) If the force each exerts on the other is 11.0 N and is repulsive, what is the charge on each?

$q_1 + q_2 = 75.0\times10^{-6}C$ $\quad |F| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1(75.0\times10^{-6}-q_1)|C^2}{(1.05)^2 m^2} = 11.0N$

a) If force is attractive, then q_1 and q_2 must be of opposite sign

$\therefore \frac{1}{4\pi\varepsilon_0}\frac{q_1(75.0\times10^{-6}-q_1)}{(1.05)^2} = -11.0$

ie $q_1^2 - 75.0\times10^{-6}q_1 - 11.0\times(1.05)^2 \times 4\pi\times 8.85\times10^{-12} = 0$

$q_1^2 - 75.0\times10^{-6}q_1 - 1.35\times10^{-9} = 0$

$q_1 = \frac{75.0\times10^{-6} \pm \sqrt{5.625\times10^{-9}+5.4\times10^{-9}}}{2}$

$= 90\times10^{-6}$ or 15×10^{-6}

$\therefore$ one charge is $\boxed{90.0\mu C, \text{ the other is } -15.0\mu C}$

b) If force is repulsive, then q_1 and q_2 must have the same sign

$\therefore \frac{1}{4\pi\varepsilon_0}\frac{q_1(75.0\times10^{-6}-q_1)}{(1.05)^2} = +11$

$\therefore q_1 = \frac{75.0\times10^{-6} \pm \sqrt{5.625\times10^{-9}-5.4\times10^{-9}}}{2}$

$= 45\times10^{-6}$ or 30×10^{-6}

$\therefore$ one charge is $\boxed{+45.0\mu C, \text{ the other is } +30.0\mu C}$

X version: a) $\frac{1}{4\pi\varepsilon_0}\frac{q_1(100\times10^{-6}-q_1)}{(1.25)^2} = -7.73$

$\therefore q_1^2 - 100\times10^{-6}q_1 - 1.344\times10^{-9} = 0$

$q_1 = \frac{100\times10^{-6} \pm \sqrt{10^{-8}+5.38\times10^{-9}}}{2}$

$= 112\times10^{-6}$ or 12×10^{-6}

$\therefore$ charges are $\boxed{+112\mu C \text{ and } -12.0\mu C}$

b) $\frac{1}{4\pi\varepsilon_0}\frac{q_1(100\times10^{-6}-q_1)}{(1.25)^2} = +7.73$

$q_1 = \frac{100\times10^{-6} \pm \sqrt{10^{-8}-5.38\times10^{-9}}}{2}$

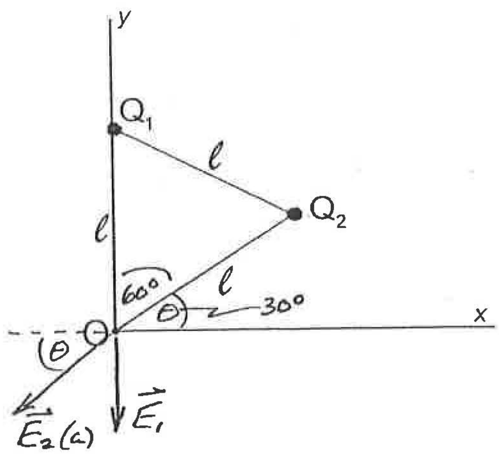
$= 84\times10^{-6}$ or 16×10^{-6}

$\therefore$ Charges are $\boxed{+84.0\mu C \text{ and } +16.}$

2. **(20 marks)** Find the total electric field **E** (as a vector) at the origin O in the figure as a result of the charges Q_1 and Q_2, under the following conditions:

10 a) The distances ℓ are 25cm, and the charges are $Q_1 = +5.0\ \mu C$ and $Q_2 = +10.0\ \mu C$;

10 b) The values of ℓ and Q_1 are the same as in part (a) but $Q_2 = -10.0\ \mu C$.



a) $\vec{E}_1 = -\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{\ell^2}\hat{j}$

$\vec{E}_2 = -\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{\ell^2}\cos\theta\,\hat{i} - \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{\ell^2}\sin\theta\,\hat{j}$

$\vec{E} = \vec{E}_1 + \vec{E}_2$

$= -\frac{1}{4\pi\varepsilon_0 \ell^2}\left[Q_2\cos\theta\,\hat{i} + (Q_1 + Q_2\sin\theta)\hat{j}\right]$

$= \frac{-9.0\times10^9\ Nm^2/C^2}{(0.25\ m)^2}\left[10.0\times10^{-6}C \times 0.866\hat{i} + (5.0\times\right.$

$\left. + 10.0\times10^{-6}C\times0.5)\hat{j}\right]$

$= \boxed{-1.24\times10^6\ N/C\ \hat{i} - 1.44\times10^6\ N/C\ \hat{j}}$

$|E| = 1.90\times10^6\ N/C \quad \phi = -131°$

b) Change the sign of Q_2

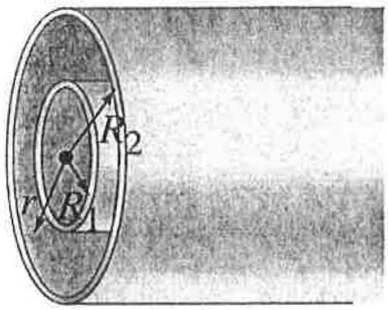
$\therefore\ \vec{E} = \frac{-9.0\times10^9 Nm^2/C^2}{(0.25m)^2}\left[-10.0\times10^{-6}C\times 0.866\hat{i} + (5.0\times10^{-6}C - 10.0\times10^{-6}C\times 0.\right.$

$= \boxed{1.25\times10^6\ N/C\ \hat{i}}$

X version: parts (a) and (b) reversed

3. **(20 marks)** A thin cylindrical shell of radius $R_1 = 3.0$ cm is surrounded by a second concentric cylindrical shell of radius $R_2 = 7.0$ cm. Both cylinders are 7.0 m long and the inner one carries a total charge $Q_1 = -4.8$ μC and the outer one $Q_2 = +5.6$ μC. The charges are uniformly distributed over the respective cylinders. For points far from the ends of the cylinders, determine the electric field at a radial distance r from the central axis for the following cases:

6 (a) 2.8 cm;
7 (b) 5.0 cm;
7 (c) 9.0 cm.



a) $E(2.8\text{ cm}) \cdot A = \frac{Q_{encl}}{\varepsilon_0} = 0$

$\therefore \boxed{E = 0}$

b) $E(5.0\text{cm}) \cdot 2\pi(0.05\text{m})\,\ell = \dfrac{\frac{-4.8\times10^{-6}\text{C}\,\ell}{7.0\text{m}}}{\varepsilon_0}$

$\therefore E = \dfrac{-4.8\times10^{-6}\text{C}}{2\pi(0.05\text{m}) \times 7.0\text{m} \times 8.85\times10^{-12}\text{C}^2/\text{Nm}}$

$\boxed{= -2.5\times10^{5}\text{ N/C (towards center)}}$

c) $E(9.0\text{cm}) \cdot 2\pi(0.09\text{m})\,\ell = \dfrac{+0.8\times10^{-6}\text{C}\,\ell}{7.0\text{m}\,\varepsilon_0}$

$E = \dfrac{+0.8\times10^{-6}\text{C}}{2\pi(0.09\text{m}) \times 7.0\text{m} \times 8.85\times10^{-12}\text{C}^2/\text{Nm}^2}$

$\boxed{= +2.3\times10^{4}\text{ N/C (pointing out)}}$

X version: a) $E(8.5\text{cm}) \cdot 2\pi(0.085\text{m})\,\ell = \dfrac{-0.8\times10^{-6}\text{C}\,\ell}{7.0\text{m}\,\varepsilon_0}$

$E = \dfrac{-0.8\times10^{-6}\text{C}}{2\pi(0.085\text{m}) \times 7.0\text{m} \times 8.85\times10^{-12}\text{C}^2/\text{Nm}^2}$

$= \boxed{-2.4\times10^{4}\text{ N/C (pointing in)}}$

b) $E(6.0\text{cm}) \cdot 2\pi(0.06\text{m})\,\ell = \dfrac{4.8\times10^{-6}\text{C}\,\ell}{7.0\text{m}\,\varepsilon_0}$

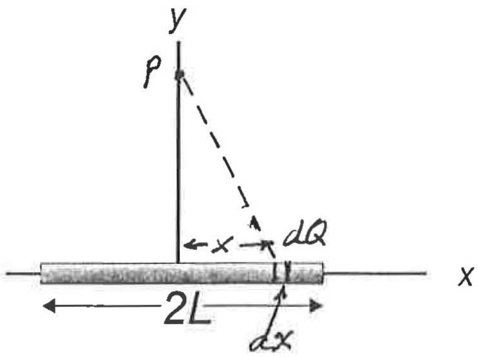
$E = \dfrac{4.8\times10^{-6}\text{C}}{2\pi(0.060\text{m}) \times 7.0\text{m} \times 8.85\times10^{-12}\text{C}^2/\text{Nm}^2}$

$= \boxed{+2.1\times10^{5}\text{ N/C (pointing out)}}$

c) No charge is enclosed

$\boxed{E = 0}$

4. **(20 marks)** A thin rod of length $2L$ is centered on the x axis as shown in the figure. The rod carries a uniformly distributed charge Q. Determine the potential V as a function of y for points along the y axis. Let $V = 0$ at infinity.



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{\lambda\, dx}{\sqrt{x^2+y^2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + \sqrt{x^2+y^2}\right)\right]_{-L}^{L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L + \sqrt{L^2+y^2}\right) - \ln\left(-L + \sqrt{L^2+y^2}\right)\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\sqrt{L^2+y^2}+L}{\sqrt{L^2+y^2}-L}\right)$$

but $\lambda = \frac{Q}{2L}$

$$\therefore \boxed{V = \frac{Q}{8\pi\varepsilon_0 L}\ln\left(\frac{\sqrt{L^2+y^2}+L}{\sqrt{L^2+y^2}-L}\right)}$$