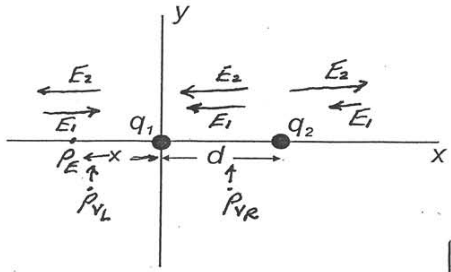
Multiple choice questions:

Question	Exam A	Exam B
MC 1	B	A
MC 2	C	B
MC 3	C	D
MC 4	A	A

1. (20 marks) The two charges designated q_1 and q_2 in the figure have the values $q_1 = -Q$ and $q_2 = +3Q/2$.

10 a) At what value(s) of x on the x axis (either positive or negative) is the electric field zero?
10 b) At what value(s) of x on the x axis (either positive or negative) is the potential zero? Let $V = 0$ at $r = \infty$.



a) Since $|q_1| < |q_2|$, the E_1 & E_2 vectors shown in figure indicate that $E_{tot} = 0$ only to the left of q_1 (at P_E). Require $E_1 = E_2$ at that point:

$$\frac{Q}{4\pi\varepsilon_0 x^2} = \frac{3Q/2}{4\pi\varepsilon_0 (x+d)^2}$$

$\therefore \frac{3}{2}x^2 = (x+d)^2$

$x^2 - 4xd - 2d^2 = 0$

$x = \frac{4d \pm \sqrt{16d^2 + 8d^2}}{2}$

$= (2+\sqrt{6})d$ (only +ve solution)

$\therefore$ $\boxed{x = 4.45d \text{ to left of } q_1}$

b) Because V is a scalar, it can be zero at two different positions on the axis, P_{V_L} & P_{V_R}.

At P_{V_L}: $\frac{Q}{4\pi\varepsilon_0 x} = \frac{3Q/2}{4\pi\varepsilon_0 (x+d)}$

$\therefore \frac{3}{2}x = (x+d) \Rightarrow \frac{1}{2}x = d$

$\therefore$ $\boxed{x = 2d \text{ to left of } q_1}$

At P_{V_R}: $\frac{Q}{4\pi\varepsilon_0 x} = \frac{\frac{3}{2}Q}{4\pi\varepsilon_0 (d-x)} \Rightarrow \frac{3}{2}x = (d-x) \Rightarrow \frac{5}{2}x = d \Rightarrow$ $\boxed{x = 0.4d \text{ to right of } q_1}$

B version:

a) $\frac{Q}{4\pi\varepsilon_0 x^2} = \frac{5Q/2}{4\pi\varepsilon_0 (x+d)^2}$

$\frac{5}{2}x^2 = (x+d)^2$

$3x^2 - 4xd - 2d^2 = 0$

$x = \frac{4d \pm \sqrt{16d^2 + 24d^2}}{6}$

$= \frac{1}{3}(2+\sqrt{10})d$

$\boxed{x = 1.72d \text{ to left of } q_1}$

b) $\frac{Q}{4\pi\varepsilon_0 x} = \frac{5Q/2}{4\pi\varepsilon_0 (x+d)}$ at P_{V_L}

$\frac{5}{2}x = x + d \Rightarrow \frac{3}{2}x = d$

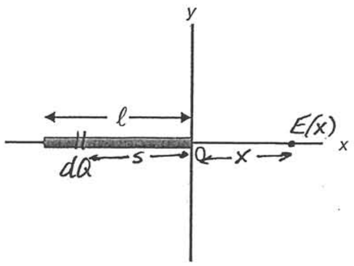
$\boxed{x = 0.67d \text{ to left of } q_1}$

$\frac{Q}{4\pi\varepsilon_0 x} = \frac{5Q/2}{4\pi\varepsilon_0 (d-x)}$

$\frac{5}{2}x = d - x \Rightarrow \frac{7}{2}x = d$

$\boxed{x = 0.29d \text{ to right of } q_1}$

2. (20 marks) A thin rod of length *l* carries a total charge Q distributed uniformly along its length. Determine the electric field along the axis of the rod starting at one end — that is, find *E(x)* for $x \geq 0$ in the figure.



$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\hat{i}$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{(s+x)^2}\hat{i}$$

$$\lambda = Q/l \qquad dQ = \lambda\,ds = Q/l\,ds$$

$$\therefore\ d\vec{E} = \frac{Q}{4\pi\varepsilon_0 l}\frac{ds}{(s+x)^2}\hat{i}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 l}\int_0^l \frac{ds}{(s+x)^2}\hat{i}$$

$$= \frac{Q}{4\pi\varepsilon_0 l}\left[-\frac{1}{s+x}\right]_0^l \hat{i}$$

$$= \frac{Q}{4\pi\varepsilon_0 l}\left[-\frac{1}{l+x}+\frac{1}{x}\right]\hat{i}$$

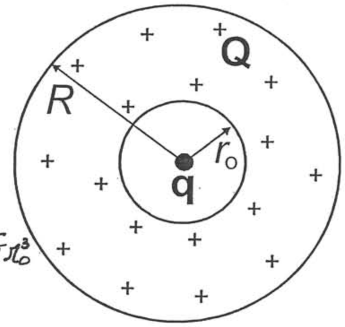
$$= \frac{Q}{4\pi\varepsilon_0 l}\frac{l}{x(l+x)}\hat{i}$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0 x(l+x)}\hat{i}}$$

B version: Same except for
y instead of x

3. **(20 marks)** A non-conducting sphere of radius R has a spherical cavity of radius r_o centered at the sphere's center (see figure). There is a charge Q distributed uniformly in the non-conducting "shell" (between $r = r_o$ and $r = R$). In addition, there is a point charge q located at the center of the cavity. Determine the electric field for

5 a) $0 < r < r_o$;
10 b) $r_o < r < R$; and
5 c) $r > R$



a) $EA = q/\varepsilon_0$

$E \cdot 4\pi r^2 = q/\varepsilon_0$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2} \qquad r < r_0}$$

b) Volume charge density $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r_0^3}$

$$EA = \frac{q + \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_0^3\right)}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q + Q\left(\dfrac{\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_0^3}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r_0^3}\right)}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0 r^2}\left[q + \frac{Q(r^3 - r_0^3)}{(R^3 - r_0^3)}\right] \qquad r_0 < r < R}$$

c) $EA = \dfrac{q + Q}{\varepsilon_0}$

$E\, 4\pi r^2 = \dfrac{q + Q}{\varepsilon_0}$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q + Q}{r^2} \qquad r > R}$$

B version a) same

b) $$\boxed{E = \frac{1}{4\pi\varepsilon_0 r^2}\left[q - \frac{Q(r^3 - r_0^3)}{(R^3 - r_0^3)}\right] \qquad r_0 < r < R}$$

c) $$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q - Q}{r^2} \qquad r > R}$$

4. (20 marks) Two identical +12.5-μC point charges are initially spaced 7.0 cm from each other. They have identical masses of 2.0 mg.

15 a) If both are released at the same instant from rest, how fast will they be moving when they are very far away from each other?

5 b) If only one is released, what will be its speed when it is very far away?

a) The potential energy of the two charges is

$$U_i = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}} = 9.0\times10^{+9}\,\frac{Nm^2}{C^2}\,\frac{(12.5\times10^{-6}C)^2}{0.070\,m}$$

$$= 20.09\,J$$

(If only part b) is solved, mark it out of 15.)

$$U_i + K_i^{\nearrow 0} = U_f^{\nearrow 0} + K_f$$

$$\therefore K_f = 20.09\,J = 2\times\tfrac{1}{2}mv^2$$ (since two are moving)

$$\therefore 20.09\,J = 2\times10^{-6}\,kg\; v^2$$

$$\therefore v = \sqrt{\frac{20.09\,kg\frac{m^2}{s^2}}{2\times10^{-6}\,kg}}$$

$$\boxed{v = 3.17\times10^3\,m/s}$$

b) In this case, only one body is moving, so

$$20.09\,J = \tfrac{1}{2}\times2\times10^{-6}\,kg\; v^2$$

$$v = \sqrt{\frac{20.09\,kg\,m^2/s^2}{1\times10^{-6}\,kg}}$$

$$\boxed{v = 4.48\times10^3\,m/s}$$

B version: a)

$$U_i = 9.0\times10^9\,\frac{Nm^2}{C^2}\,\frac{(8.5\times10^{-6}C)^2}{0.035\,m}$$

$$= 18.58\,J$$

$$v = \sqrt{\frac{18.58\,kg\,m^2/s^2}{3\times10^{-6}\,kg}}$$

$$\boxed{v = 2.49\times10^3\,m/s}$$

b)

$$v = \sqrt{\frac{18.58\,kg\,m^2/s^2}{1.5\times10^{-6}\,kg}}$$

$$\boxed{v = 3.52\times10^3\,m/s}$$