Short Answers:

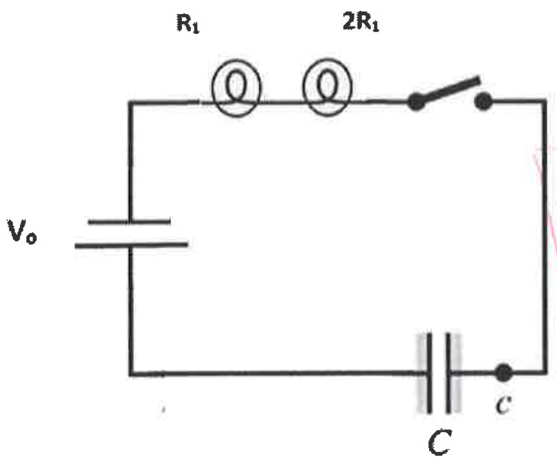
A) Two light bulbs with resistances R_1 and $2R_1$ are in series with a capacitor C and a battery with voltage V_o. At time $t = 0$ a switch S is flipped to close the circuit.

i) The current drops to half of its initial value, what are the potential differences across resistor R_1 and across resistor $2R_1$?

At $t=0 \quad V_c = 0 \Rightarrow V_o - I(3R_1) = 0 \Rightarrow I = \frac{V_o}{3R_1}$

When $I = \frac{V_o}{6R_1}$ $\boxed{V_{R_1} = \frac{V_o}{6}, \quad V_{2R_1} = \frac{V_o}{3}}$

ii) How much energy is stored in the capacitor at that instant?



$V_c = V_o - \frac{V_o}{3} - \frac{V_o}{6} = \frac{V_o}{2}$

OR $e^{-\frac{t}{\tau}} = \frac{1}{2} \Rightarrow V_c = V_o(1-\frac{1}{2})$, $V_c = \frac{V_o}{2}$

$U = \frac{1}{2}CV^2 = \frac{1}{2}C\frac{V_o^2}{4} = \boxed{\frac{1}{8}CV_o^2}$

LO	S	U
40.1		
42.1		
44.1		
30.1		
~~30.1~~ 36.1		

B) A long conducting wire lies along the z-axis and carries a current I_o, along the positive z-direction. A length L of the wire is enclosed by a cylindrical shell of radius R whose central axis is parallel to the z-axis but shifted a distance $R/2$ along the x-axis.

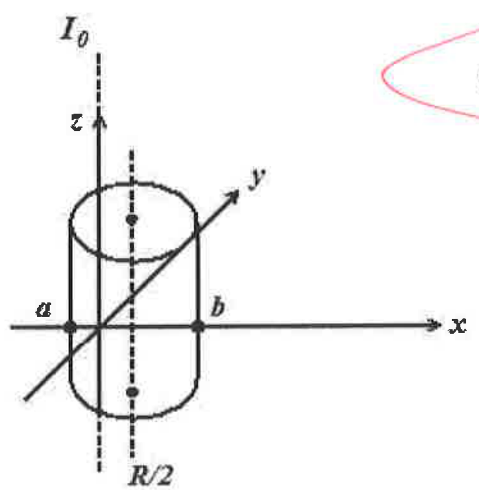
i) What are the magnetic field vectors $\vec{B}_a$ and $\vec{B}_b$ at the locations a and b shown in the figure?

$B = \frac{\mu_o I}{2\pi r}$

$B_a = \frac{\mu_o I}{2\pi(\frac{R}{2})} = \frac{\mu_o I}{\pi R}$ out of ~~screen~~ paper OR $\vec{B}_a = -\frac{\mu_o I}{\pi R}\hat{j}$

$B_b = \frac{\mu_o I}{2\pi(\frac{3R}{2})} = \frac{\mu_o I}{3\pi R}$ into ~~screen~~ paper OR $\vec{B}_b = \frac{\mu_o I}{3\pi R}\hat{j}$

ii) If the current is doubled, does the magnetic flux through the cylinder increase, decrease, or remain constant? Explain.



$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$

since $\vec{B}$ is tangential to surface, i.e $\perp$ to $d\vec{A}$

$\Rightarrow \Phi_B$ stays constant if current is doubled.

LO	S	U
1.1		
56.1		
57.1		
58.1		

C) The figure shows four Gaussian surfaces surrounding a distribution of charges. For the following questions, choose from Gaussian surfaces a, b, c, d.

i) Which Gaussian surfaces have a total electric flux of $+q/\epsilon_o$ through them?

$$\Phi_E = \frac{Q_{enc}}{\varepsilon_o}$$

For $\boxed{\text{surface } b,}$ $\Phi_E = +q/\varepsilon_o$

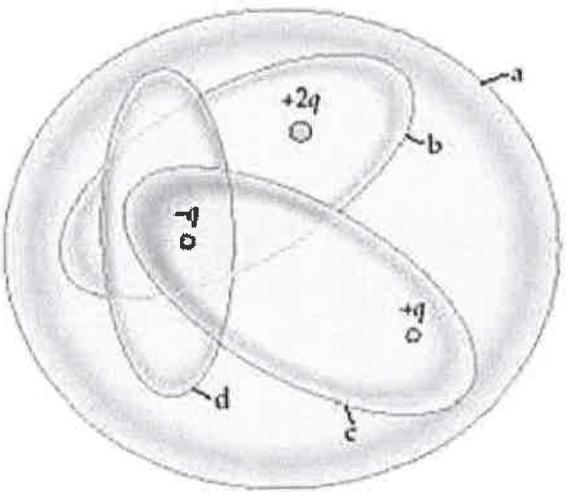
a: $Q_{enc} = +2q$

b: $Q_{enc} = +q$

c: $Q_{enc} = 0$

d: $Q_{enc} = -q$

ii) Which Gaussian surfaces have no total electric flux through them?



For $\boxed{\text{surface } c,}$ $\Phi_E = 0$

LO	S	U
16.1		
17.1		
16.2		
17.2		

D) Three positive point charges, Q_1, Q_2 and Q_3, are located on the corners of a right triangle as shown in the figure.

i) Find the force (magnitude and direction) exerted on charge Q_3 due to the other two charges.

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$F_{13,x} = \frac{kQ_1Q_3}{(5L)^2}\left(\frac{4L}{5L}\right)$$

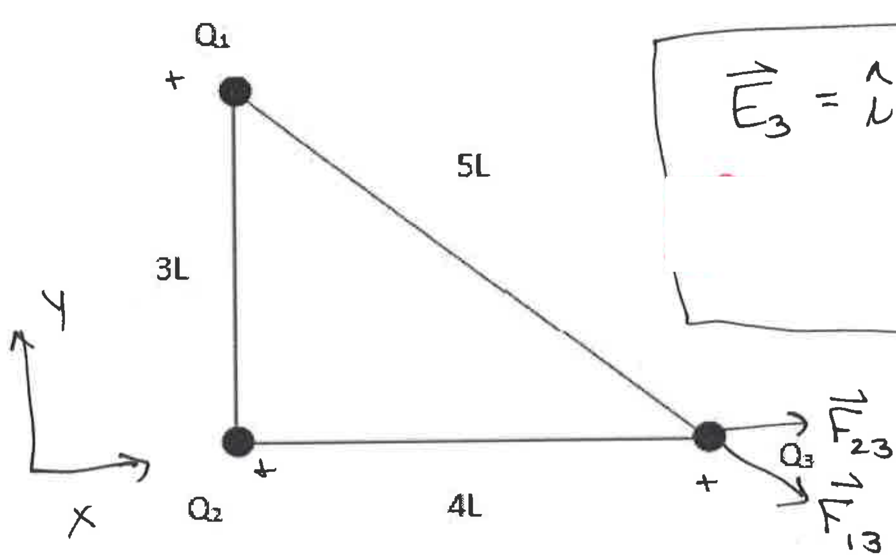
$$F_{13,y} = -\frac{kQ_1Q_3}{(5L)^2}\left(\frac{3L}{5L}\right)$$

$$F_{23,x} = \frac{KQ_2Q_3}{(4L)^2}$$

$$F_{23,y} = 0$$

$$\Rightarrow \vec{F}_3 = \hat{i}\,KQ_3\left[\frac{Q_1}{(5L)^2}\cdot\frac{4}{5} + \frac{Q_2}{(4L)^2}\right] - \hat{j}\,KQ_3\cdot\frac{Q_1}{(5L)^2}\cdot\frac{3}{5}$$

ii) What is the value of the electric field (magnitude and direction) at the location of charge Q_3?



$$\boxed{\vec{E}_3 = \hat{i}\,K\left[\frac{Q_1}{(5L)^2}\frac{4}{5} + \frac{Q_2}{(4L)^2}\right] - \hat{j}\,K\frac{Q_1}{(5L)^2}\frac{3}{5}}$$

LO	S	U
1.2		
2.1		
10.1		
1.3		
2.2		
12.1		

E) A circular loop of wire of radius, r, lies in the xy plane and is subject to a spatially uniform but time-changing magnetic field,

$B(t) = (5.0\text{ T} - (0.1\text{ T/s}^2)\, t^2)$ in the z-direction.

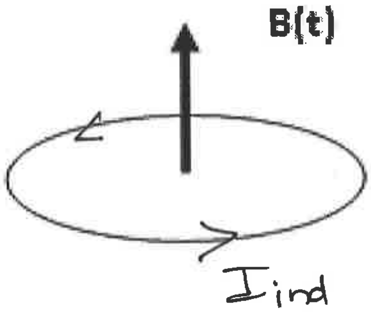
i) Find the induced **EMF** produced in this loop.

$$\mathcal{E} = -\frac{d\Phi_E}{dt} \qquad \Phi_E = \int \vec{B}\cdot d\vec{A} = [5.0 - 0.1t^2]\pi r^2$$

$$\frac{d\Phi_E}{dt} = -0.2t \cdot \pi r^2 \qquad |\mathcal{E}| = (0.2t)\pi r^2$$

ii) Indicate in the figure the direction that the induced current will flow under these circumstances.

→ flux is decreasing



⇒ current induced is counter clockwise

LO	S	U
58.2		
59.1		
60.1		

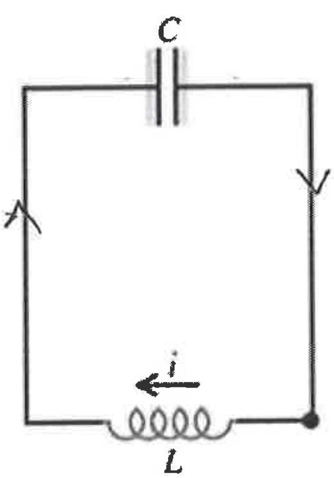
F) At $t = 0$ in the LC circuit shown below, the capacitor is uncharged and there is a current of $I_{\max}$ flowing clockwise in the circuit.

i) Find the maximum value of the charge that will appear on the capacitor.

$$E = \frac{1}{2}LI_{max}^2 = \frac{1}{2}\frac{Q_{max}^2}{C}$$

$$\Rightarrow Q_{max} = \sqrt{LC}\, I_{max}$$

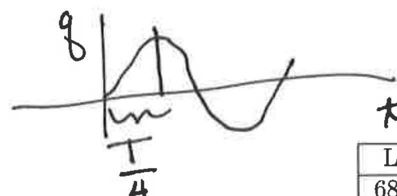
ii) How long after $t = 0$ does it take for the capacitor to reach this value of charge for the first time?



$$q = Q_{max}\sin(\omega t)$$

$$q(0) = 0$$

$$\Downarrow$$



at $t = \frac{T}{4}$, $|q| = Q_{max}$

$$\Rightarrow t = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega} \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow t = \frac{\pi\sqrt{LC}}{2}$$

LO	S	U
68.1		
30.2		
70.1		

G) The current is changing as a function of time through an unknown inductor. When the current through the inductor is decreasing at a rate of 5.0 A/s, the voltage across the inductor is measured to be 20.0 V.

i) In terms of the quantities given, what is the value of the self inductance of this inductor?

when $\frac{di}{dt} = -5.0\,\frac{A}{s}$, $V_L = 20\,V$

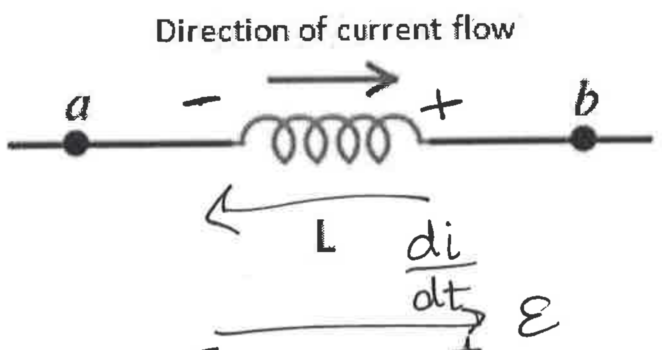
$V_L = -L\frac{di}{dt}$ ← 67

$20\,V = L\left(5\,\frac{A}{s}\right)$

$\Rightarrow \boxed{L = 4\,H}$

ii) Indicate in the sketch which side of the inductor will be at the higher potential under these conditions.

LO	S	U
67.1		
60.2		



H) The electric field components of a propagating electromagnetic wave are shown in the figure below at time t=0. The electric field is given by $\vec{E}(z,t) = E_o\, cos(kz - \omega t)\hat{j}$ at an arbitrary point in time and space along the line of propagation. The direction of propagation is labeled **v**.

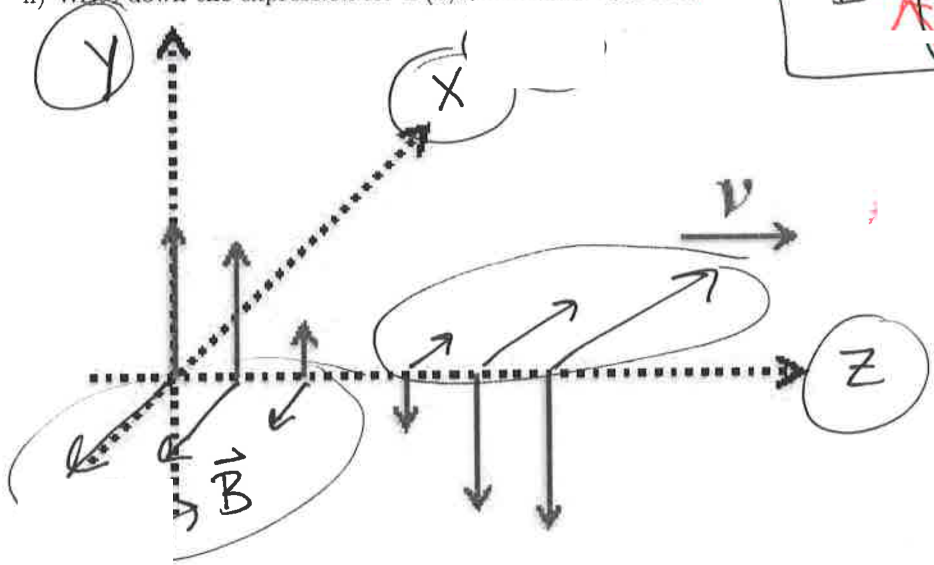
i) On the figure below, label the x,y and z axes. Draw the magnetic field components of the electromagnetic wave at the same points where the electric field is shown.

75 direction along +x

+76

ii) Write down the expression for $\vec{B}(z,t)$ in vector notation.

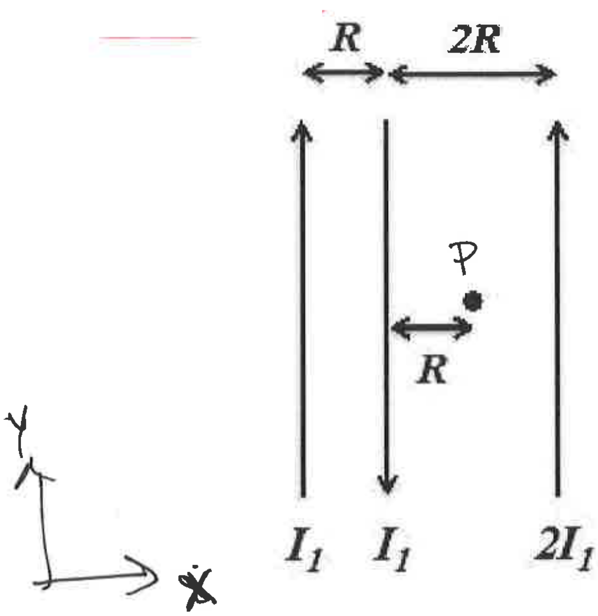
$\boxed{\vec{B} = -\frac{E_o}{c}\cos(kz-\omega t)\,\hat{i}}$



LO	S	U
75.1		
76.1		
75.2		
1.4		
76.2		

Prob 1 Three very long current-carrying wires are aligned parallel with each other in the x-y plane. The left wire carries a current I_1 upward, the middle wire carries a current I_1 downward, and the right wire carries a current $2I_1$ upward.

(a) What is the magnitude and direction of the magnetic field halfway between the middle and right wires?



At pt. P

$$\vec{B}_{left} = \frac{\mu_0 I_1}{2\pi(2R)} \text{ into page}$$

$$\vec{B}_{middle} = \frac{\mu_0 I_1}{2\pi R} \text{ out of page}$$

$$\vec{B}_{right} = \frac{\mu_0 (2I_1)}{2\pi R} \text{ out of page}$$

$$\vec{B}_{Net} = \frac{\mu_0 I_1}{\pi}\left[\frac{1}{4R} - \frac{1}{2R} - \frac{1}{R}\right] \text{ into page}$$

$$= \frac{-5\mu_0 I_1}{4\pi R} \text{ into page}$$

$$\text{OR } \boxed{+\frac{5\mu_0 I_1}{4\pi R} \text{ out of page}}$$

(b) What is the net force per unit length on the middle wire due to the other wires?

$$\vec{F}_{middle} = I_{middle}\vec{L}_{middle} \times \vec{B}_{left} + I_{middle}\vec{L}_{middle} \times \vec{B}_{right}$$

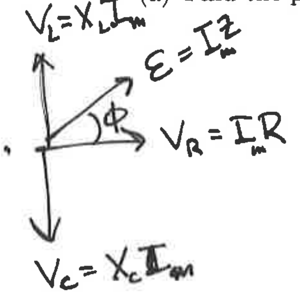
$$\Rightarrow \frac{\vec{F}}{L} = I_1\left(\frac{\mu_0 I_1}{2\pi R}\right)\hat{i} - I_1\left(\frac{\mu_0 I_1}{2\pi R}\right)\hat{i}$$

$$\boxed{\frac{\vec{F}}{L} = 0}$$

LO	S	U
2.3		
3.1		
48.1		
56.2		
57.2		
3.2		
55.1		

Prob 2 In the RLC circuit shown, the generator voltage can be represented by $\mathcal{E}(t) = \mathcal{E}_{\max} \, cos(\omega t + \phi)$. The values of the resistance R, peak generator voltage $\mathcal{E}_{\max}$, generator frequency ω and phase angle ϕ by which the generator EMF leads the current are known values (**L and C are not known**).

(a) Find the peak current, $I_{\max}$, through this circuit in terms of the known values.



$$\cos\phi = \frac{R}{Z} \Rightarrow Z = \frac{R}{\cos\phi}$$

$$\mathcal{E}_{max} = I_{max} Z \Rightarrow I_{max} = \frac{\mathcal{E}_{max}}{R/\cos\phi} = \frac{\mathcal{E}_{max}\cos\phi}{R}$$

(b) What is the average power input for this circuit (again, in terms of the known values)?

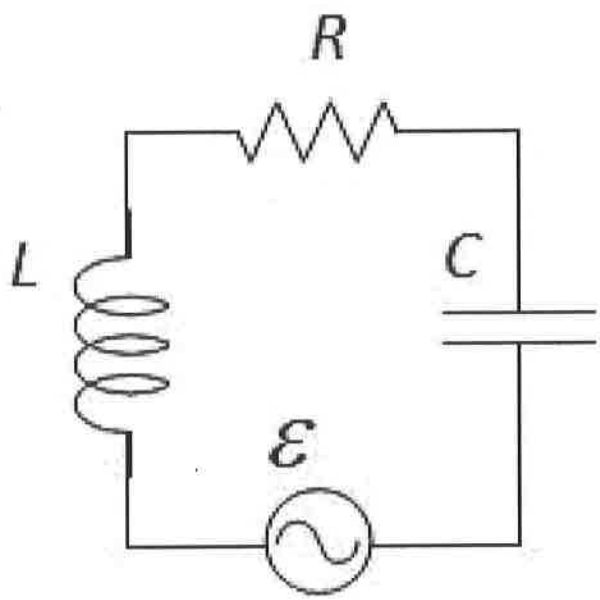
$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos\phi = \frac{\mathcal{E}_{max} I_{max}}{2} \cos\phi$$

$$= \frac{\mathcal{E}_{max}^2 \cos^2\phi}{2R}$$

(c) What would be the maximum current if the frequency, ω, is set to ω_o the resonant frequency?

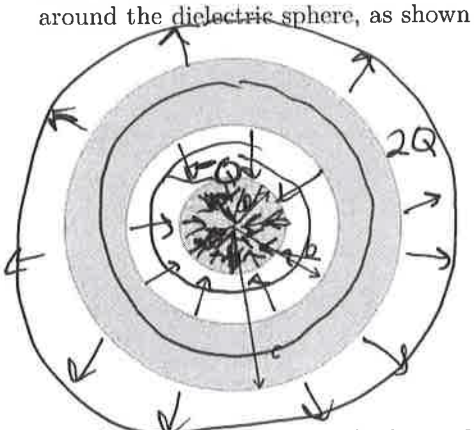
if $\omega = \omega_o$ $\cos\phi = 1 \Rightarrow Z = R$

$$\Rightarrow I_{max} = \frac{\mathcal{E}_{max}}{R}$$



LO	S	U
72.1		
3.3		
74.1		
73.1		

Prob 3 A dielectric sphere of radius a is uniformly charged with a negative electric charge $(-Q)$. A spherical *conducting metal shell* of internal radius b and external radius c is positively charged with a charge $2Q$ and is wrapped concentrically around the dielectric sphere, as shown in the figure.



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

(a) Calculate the electric field as a function of distance r from a center of the sphere for the following regions: (i) $a < r < b$, (ii) $b < r < c$, (iii) $r > c$.

$a<r<b$:

$$E(4\pi r^2) = \frac{-Q}{\varepsilon_0}$$

$$E = \frac{-Q}{4\pi\varepsilon_0 r^2}$$

$b<r<c$:

$$E = 0$$

$r>c$:

$$E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

(b) Sketch the electric field lines in all these four regions in the figure. In this same figure sketch at least 3 equipotential surfaces.

(c) Calculate the surface density of electric charge on the inner and outer surfaces of the metal shell.

$$\sigma_b = \frac{+Q}{4\pi b^2} \qquad \sigma_c = \frac{+Q}{4\pi c^2}$$

(d) Suppose a point positive charge q is placed at the distance $2c$ from the center of dielectric sphere. Find the work done **by the net electric field** when this charge moves to the new position at the distance $3c$ from the center of dielectric sphere.

$$W = -\Delta U = -q\Delta V = -q(V_f - V_i)$$

$$= -q(V(3c) - V(2c)) = -q\left[-\int_{r_i}^{r_f} \vec{E}\cdot d\vec{r}\right]$$

$$= +q\int_{2c}^{3c} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= -\frac{qQ}{4\pi\varepsilon_0 r}\Bigg|_{2c}^{3c} = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{1}{2c} - \frac{1}{3c}\right]$$

LO	S	U
18.1		
19.1		
8.1		
19.2		
20.1		
19.3		
14.1		
25.1		
8.2		
20.2		
3.4		
6.1		
22.1		
26.1		