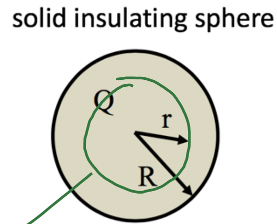
1) [*8 pts*] **Gauss' Law**. A solid insulating sphere of radius R is uniformly charged with total charge Q and placed at the origin. Use Gauss' law to determine the electric field for a radius r < R (i.e. inside the sphere) :



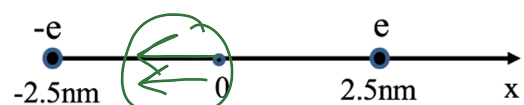
$Q_{in} = Q\left(r^3/R^3\right)$

$E(\underbrace{\text{area}}_{4\pi r^2}) = Q_{in}/\epsilon_0$

$\rightarrow E = \frac{Q}{4\pi\epsilon_0}\left(\frac{r}{R^3}\right)$

(A) E(r) = kQ/r^2
(B) E(r) = kQr/R^3 correct
(C) E(r) = kQr/2R^3
(D) E(r) = kQ/rR
(E) E(r) = kQ/2r^2
(F) E(r) = kQr^2/R^4

2) [*8 pts*] **Electric Field**. An electron and a proton are separated by a distance of 5nm (see figure). The electric field at their midpoint is :

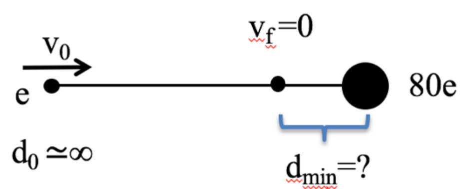


fields add

$E = 2 \times \left(\frac{kQ}{r^2}\right)$ $Q = 1.6\times10^{-19}$ C, $r = 2.5\times10^{-9}$ m

(A) 0
(B) $2.3\text{x}10^{6}$ N/C in the direction of the electron
(C) $2.3\text{x}10^{6}$ N/C in the direction of the proton
(D) $1.15\text{x}10^{8}$ N/C in the direction of the electron
(E) $1.15\text{x}10^{8}$ N/C in the direction of the proton
(F) $4.6\text{x}10^{8}$ N/C in the direction of the electron correct
(G) $4.6\text{x}10^{8}$ N/C in the direction of the proton

3) [*6 pts*] **Electric Potential Energy**. A proton (charge e and mass m) is approaching a stationary mercury nucleus (charge 80e) head on. When it is far away, the proton's speed is v_0. What is the distance of closest approach d_{min} of the proton to the nucleus ?



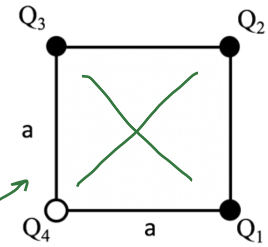
$v_f = 0 \rightarrow$ all KE $\rightarrow$ PE

$\frac{1}{2} m v_0^2 = \frac{ke(80e)}{r}$

$r = \frac{ke(160e)}{m v_0^2}$

(A) 0
(B) $[k\,e / (mv_0)]^{1/2}$
(C) $80\,k\,e / (mv_0)$
(D) $2\,k\,e^2 / (mv_0^2)$
(E) $[\,80\,k\,e^2 / (mv_0^2)\,]^{1/2}$
(F) $160\,k\,e^2/(mv_0^2)$ correct

4) [*8 pts*] **Electric Potential**. Four charges of equal magnitude are placed on the corners of a square of sidelength a. Three of the charges are positive ($Q_{1,2,3} = Q > 0$) and 1 is negative ($Q_4 = -Q$). The total potential energy of this configuration (with the convention $U(r \to \infty) = 0$) is :



(A) $4\ kQ^2/a$
(B) $2\ kQ^2/a$
(C) kQ^2/a
(D) 0 correct
(E) $-2\ kQ^2/a$
(F) $-4\ kQ^2/a^2$

$U = \sum_{pairs} \frac{kq_iq_j}{r}$

(4 outer edges cancel)
(2+ & 2- terms)

$= 0$

also diagonals cancel
(also equal/opposite)

5) [*6 pts*] **Energy in Capacitor**. An air-filled insulated parallel-plate capacitor is held at a fixed charge. If the separation of its plates is doubled, the electric energy stored in the capacitor is :

(A) ¼ of the original
(B) ½ of the original
(C) unchanged
(D) doubled correct
(E) quadrupled

$U = \frac{Q^2}{2C}$ $C = \frac{\varepsilon_0 A}{d}$

6) [*8 pts*] **Electric Power**. A light bulb has a power output of 60W. The bulb is connected to a 24 V battery. The current drawn from the battery and the resistance of the light bulb are :

(A) 0.2 A and 0.104 Ω
(B) 0.2 A and 1.4 Ω
(C) 0.4 A and 0.104 Ω
(D) 0.4 A and 240 Ω
(E) 2.5 A and 0.104 Ω
(F) 2.5 A and 9.6 Ω correct

$IV = 60W \to I = \frac{60W}{24V} = 2.5A$

$R = \frac{V}{I} = \frac{24V}{2.5A} = 9.6\,\Omega$

7) [*6 pts*] **Discharging RC Unit**. Consider a simple RC circuit with an initially charged capacitor of capacitance C = 1mF and unknown resistance R. When closing the switch, the current drops to 10% of its initial value within a time of 1.8s. The resistance in this circuit is :

(A) 53 Ω
(B) 140 Ω
(C) 482 Ω
(D) 783 Ω correct
(E) 1280 Ω
(F) 4145 Ω

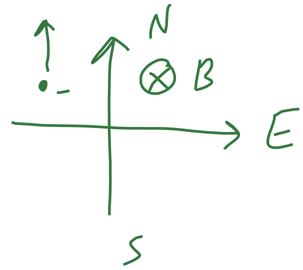
V & I decay as $e^{-t/RC}$

$e^{-(1.8s)/RC} = 0.1$

$1.8s = (RC)\ln 10$

$R = \frac{1.8s}{1mF \cdot \ln 10}$

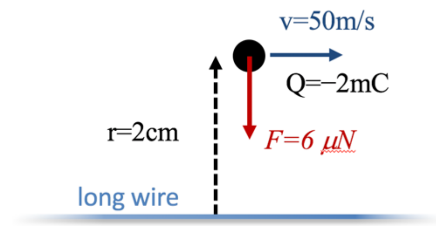
8) [*6 pts*] **Lorentz Force**. A laboratory has its four corridors marked with the north, south, east and west directions. It is in a uniform magnetic field that points downward (into the ground); there is no electric field. A negatively charged particle moves north. The Lorentz force on the particle points to :



use $q\vec{v}\times\vec{B}$ with right hand rule

(A) North
(B) South
(C) East (correct)
(D) West
(E) Up
(F) Down

9) [8 *pts*] **Magnetic Force from Current**. A long straight wire carries an unknown current I. A negatively charged particle (Q =-2mC) at 2 cm from the wire has a velocity of 50m/s along the direction of the wire (see figure). The particle experiences a force of 6 μN towards the wire. Determine the magnitude and direction of the current.



$F = Q\,v\,B$, B must point into page so I to left by R.H.R.

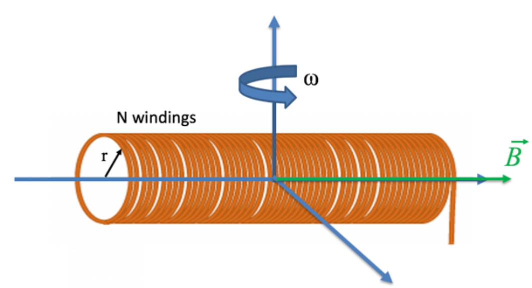
Q: 2 mC

$B = \dfrac{\mu_0 I}{2\pi(0.02\,m)}$

↳ solve for I

(A 1.91 A along the electron's velocity
(B) 1.91 A opposite the electron's velocity
(C) 6 A along the electron's velocity
(D) 6 A opposite the electron's velocity correct
(E) 18.8 A along the electron's velocity
(F) 18.8 A opposite the electron's velocity

10) [*6 pts*] **Magnetic Induction**. To determine the strength B of a uniform magnetic field in his laboratory, an engineer uses a solenoid (a long magnetic coil of radius **r** with **N** windings), rotating it at an angular frequency $\omega = 2\pi f$ about an axis perpendicular to the magnetic field and to the symmetry axis along the solenoid. He detects an induced EMF with a maximal value of ε. The magnetic field strength is :



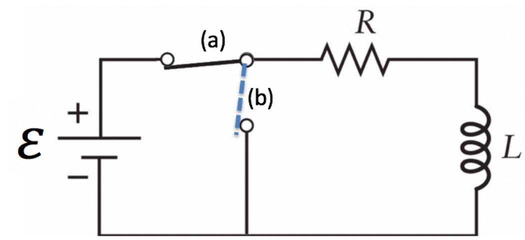
(A) $B = \varepsilon / N\pi r^2\omega$ correct
(B) $B = \varepsilon\omega / N\pi r^2$
(C) $B = \varepsilon\pi r^2 / N\omega$
(D) $B = 2\pi r / N\omega\varepsilon$
(E) $B = N\varepsilon / 2\pi r\omega$
(F) $B = \varepsilon / N 2\pi r\omega$

with $\Phi = N\pi r^2 B\cos\omega t$,

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\underbrace{\omega N\pi r^2 B}_{\text{max value}}\sin\omega t$$

$$\hookrightarrow B = \frac{\mathcal{E}}{\omega N\pi r^2}$$

11) [*8 pts*] **RL Unit**. A resistor (resistance R) and an inductor (inductance L) are connected in series to a battery with EMF ε, with switch initially at position (a) (see figure). At t=0, the switch is flipped to position (b) to remove the battery from the circuit (see figure), and the current starts to drop off as $I(t) = I_0 \exp(-t/\tau)$. The magnitude of the voltage across the inductor at t=0 and after a long time are :



(A) 0 and 0
(B) 0 and ε
(C) 0 and RI_0
(D) RI_0 and 0 correct
(E) RI_0 and RI_0
(F) ε and RI_0

t=0 Inductor same $|\Delta V|$ as resistor by loop rule

$t \Rightarrow \infty$ Inductor $L\frac{dI}{dt} = 0$

↑ I unchanging

12) [*6 pts*] **Displacement Current**. An air-filled parallel-plate capacitor is being charged leading to an increase of its electric field from 0 to 12000 V/m over a time interval of 1.5 s. Compute the displacement current through a $(25\ \text{cm})^2$ area parallel to the plates inside the capacitor.

(A) 1.8×10^{-10} A
(B) 4.4×10^{-9} A correct
(C) 1.7×10^{-9} A
(D) 1.0×10^{-8} A
(E) 0.33×10^{-7} A
(F) 0.33×10^{-5} A

$$I_D = \varepsilon_0 \frac{d}{dt}(EA)$$
$$= 8.85 \times 10^{-12}\,\frac{F}{m} \cdot \left(\frac{12000\ V/m}{1.5\ s}\right)(0.25\ m)^2$$
$$=$$

13) [*8 pts*] **EM Plane Wave**. The electric field of a plane wave is described by $\vec{E} = E_0 \sin(ky-\omega t)\,\hat{k}$. The velocity and the magnetic field of this wave are oriented in :

+y direction

(A) x- and y-direction
(B) x- and z-direction
(C) y- and x-direction correct
(D) y- and z-direction
(E) z- and x-direction
(F) z- and y-direction

$\hat{E} \times \vec{B}$ along $+\hat{y} \Rightarrow B \sim +\hat{x}$

14) [*8 pts*] **Energy in EM wave**. An electromagnetic wave has a B-field of amplitude $2 \cdot 10^{-6}$ T. The wave irradiates an area of $0.5\ m^2$. Calculate the intensity of the wave and the time it takes to deposit 1 kJ onto the irradiated area.

(A) 1.59 x 10^{-6} W/m^2 and 1.3 x 10^8 s
(B) 1.59 x 10^{-6} W/m^2 and 25 s
(C) 477 W/m^2 and 4.2 s correct
(D) 477 W/m^2 and 1.1 s
(E) 1.3 x 10^5 W/m^2 and 0.015 s
(F) 1.3 x 10^5 W/m^2 and 0.12 s

$$I = \frac{1}{2}\varepsilon_0 c E_0^2$$ use $E_0 = B_0 c$

$$= \frac{1}{2}(8.85\times10^{-12})\frac{F}{m}(c^3)B_0^2$$

$3\times10^8 \frac{m}{s}$ $2\,\mu T$

$$= 477\ W/m^2$$

$$U = 1\,kJ = 477\frac{W}{m^2}(0.5\,m^2)(\Delta t)$$

$$\hookrightarrow \Delta t \sim 4.2\ s$$