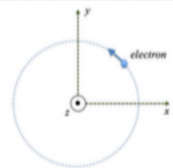
1)

[10 pts.] An electron (charge −e, mass m) moves at constant speed on a circular orbit in the x-y-plane, counter-clockwise with a period T, in a uniform magnetic field (see figure). The magnetic field is (hint: the centripetal force is $F=mv^2/r$)



$\vec{B} = +\frac{m}{eT}\hat{z}$

$$R = \frac{mv}{qB} \; \& \; T = \frac{2\pi R}{v}$$

$\vec{B} = -\frac{m}{eT}\hat{z}$

$$\Downarrow$$

$\vec{B} = +\frac{mT}{2\pi e}\hat{z}$

$$B = \frac{mv}{q\left(Tv/2\pi\right)}$$

$\vec{B} = -\frac{mT}{2\pi e}\hat{z}$

$\vec{B} = +\frac{2\pi m}{eT}\hat{z}$

$\vec{B} = -\frac{2\pi m}{eT}\hat{z}$

$\vec{B} = +\frac{mT}{e}\hat{z}$

$\vec{B} = -\frac{mT}{e}\hat{z}$

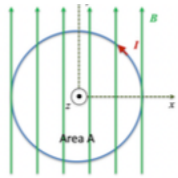
$\vec{B} = +\frac{m}{2\pi eT}\hat{z}$

$\vec{B} = -\frac{m}{2\pi eT}\hat{z}$

None of the above

2)

2) *[10 pts.]* A current loop of area $0.1m^2$, carrying a counter-clockwise current of 0.5 A, is lying in the x-y plane (see figure). A uniform magnetic field of magnitude B = 0.25 T is pointing in the positive y-direction. Using the magnetic moment of the loop, compute the torque on the current loop.



$\vec{\tau} = \vec{\mu} \times \vec{B}$

$\Downarrow$

$|\tau| = IAB$

$= 0.0125 \, Nm$

cross product $\Rightarrow -\hat{x}$

= + 0.25 $N{*}m\,\hat{x}$

= − 0.25 $N{*}m\,\hat{x}$

= + 0.13 $N{*}m\,\hat{y}$

= − 0.13 $N{*}m\,\hat{y}$

= − 0.013 $N{*}m\,\hat{x}$

= + 0.26 $N{*}m\,\hat{y}$

= − 0.013 $N{*}m\,\hat{y}$

= + 0.05 $N{*}m\,\hat{x}$

= − 0.05 $N{*}m\,\hat{x}$

None of the above

3)

3) [*6 pts.*] The work done by the Lorentz force acting on and electron moving in a uniform magnetic field (no electric field)

depends on the direction of motion of the electron

is positive, keeping it on a helical (or circular) path

is negative, keeping it on a helical (or circular) path

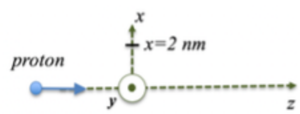
keeps changing sign during the electrons motion

is zero

None of the above

4)

4) [*10 pts.*] A proton (charge +e) is moving along the z-axis in positive direction. When it passes through z=0, it produces a magnetic field of 0.5 T at x = 2 nm. What is the direction of the magnetic field and the speed of the proton?



B in positive z-direction, speed 3.25 * 10^8 m/s

B in negative z-direction, speed 3.25 * 10^8 m/s

B in positive y-direction, speed 1.25 * 10^8 m/s

B in negative y-direction, speed 1.25 * 10^8 m/s

B in positive y-direction, speed 0.25 * 10^8 m/s

B in negative y-direction, speed 0.25 * 10^8 m/s

B in positive x-direction, speed 0.025 * 10^8 m/s

B in negative x-direction, speed 0.025 * 10^8 m/s

None of the above

$+\hat{y}$ right-hand rule;

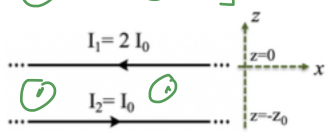
$$|B| = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

$$v = \frac{(0.5T)(2\times10^{-9}m)^2}{10^{-7}\frac{Tm}{A}\cdot 1.6\times10^{-19}C}$$

$$= 1.25\times10^{8}\ m/s$$

5)

5) *[8 pts.]* Two straight long parallel wires in x-direction are placed in the x-z-plane carrying current $I_1 = 2\ I_0$ and $I_2 = I_0$, see the figure below. Wire-1 is located at z=0 and wire-2 is at $z=-z_0$ ($z_0>0$). The total magnetic field is zero at :



B_1 B_2 here $|B_1| > |B_2|$ always

B_1 B_2 can cancel,

$z = -4\ z_0$

$z = -2\ z_0$

$z = -\frac{2}{3}\ z_0$

$z = -z_0/2$

$z = z_0$

$z = 2\ z_0$

$z = 4\ z_0$

nowhere

None of the above

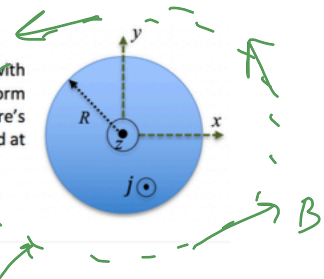
$$0 = \frac{\mu_0 (2I_0)}{2\pi |z|} - \frac{\mu_0 (I_0)}{2\pi (|z| - z_0)}$$

$$\frac{|z|}{2} = |z| - z_0$$

$$\hookrightarrow \quad |z| = 2z_0$$

6)

[10 pts.] A long uniform cylindrical wire of radius R is aligned with and centered on the z-axis as shown in the figure. It carries a uniform current density j in the direction out of the page. Apply Ampère's law to compute the direction and magnitude of the magnetic field at any location which is a distance r > R from the wire.



$B \oint dl$

$$B \times 2\pi r = \mu_0 (J\pi R^2)$$

$$B = \frac{\mu_0 J R^2}{2r}$$

$B = \frac{\mu_0}{2}\left(\frac{j}{r R^2}\right)$ in the y-direction

$B = \frac{\mu_0}{2}\left(\frac{j}{r R^2}\right)$ in the tangential direction

$B = \frac{\mu_0}{2}\left(\frac{j}{r R^2}\right)$ in the radial direction

$B = \frac{\mu_0}{2}\left(\frac{j R^2}{r}\right)$ in the x-direction

$B = \frac{\mu_0}{2}\left(\frac{j R^2}{r}\right)$ in the tangential direction

$B = \frac{\mu_0}{2}\left(\frac{j R^2}{r}\right)$ in the radial direction

$B = \frac{\mu_0}{2}\left(\frac{j R}{r^2}\right)$ in the z-direction

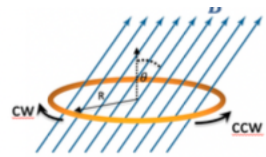
$B = \frac{\mu_0}{2}\left(\frac{j R}{r^2}\right)$ in the tangential direction

$B = \frac{\mu_0}{2}\left(\frac{j R}{r^2}\right)$ in the radial direction

None of the above

7)

7) [*8 pts.*] A current loop of radius R is located in the x-y-plane and immersed in a uniform magnetic field that makes an angle θ with the normal vector of the loop, see figure. The magnetic field strength increases at a rate dB/dt.
What is the magnitude and direction (as viewed from above) of the electric field induced along the current loop? (cw=clockwise, ccw=counter-clockwise)



$E = \frac{R}{2}\frac{dB}{dt}\cos(\theta)$, ccw

$E = \frac{R}{2}\frac{dB}{dt}\cos(\theta)$, cw

$E = R^2\frac{dB}{dt}\sin(\theta)$, ccw

$E = \frac{1}{2R}\frac{dB}{dt}\cos(\theta)$, cw

$E = \frac{1}{2R}\frac{dB}{dt}\tan(\theta)$, ccv

$E = \pi R^2\frac{dB}{dt}\sin(\theta)$, cw

$E = \frac{R}{2}\frac{dB}{dt}\tan(\theta)$, ccw

$E = \pi R^2\frac{dB}{dt}\cos(\theta)$, cw

None of the above

- CW by Lenz's law
- $\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt}(B \cdot \pi R^2 \cdot \cos\theta)$

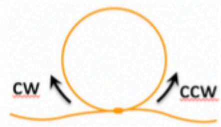
$\rightarrow \mathcal{E} = \pi R^2 \cos\theta \frac{dB}{dt}$

but $\mathcal{E} = \int E \cdot ds = E(2\pi R)$

$\rightarrow E = \frac{R\cos\theta \frac{dB}{dt}}{2}$

8)

[8 pts.] A circular current loop (initial radius r_0) is made of a movable cable that can be pulled to contract the loop, as shown on the figure. The loop is immersed in a uniform magnetic field pointing out of the page (parallel to the normal vector of the loop). A student starts to pull on both ends of the cable, reducing the radius as $r(t)=r_0-vt$. Calculate the EMF induced in the loop.



$B\,2\pi v\,(r_0 - vt)$, inducing a clockwise current

$B\,2\pi v\,(r_0 - vt)$, inducing a counter-clockwise current

$B\,\pi v\,(r_0 - vt)$, inducing a clockwise current

$B\,\pi v\,(r_0 - vt)$, inducing a counter-clockwise current

$B\,\pi\,(r_0 - vt)^2$, inducing a clockwise current

$B\,\pi\,(r_0 - vt)^2$, inducing a counter-clockwise current

$B\,2\,\pi\,v\,r_0$, inducing a clockwise current

$B\,2\,\pi\,v\,r_0$, inducing a counter-clockwise current

None of the above

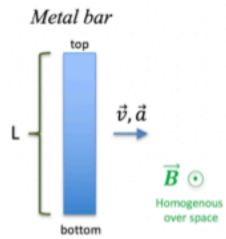
$$\Phi = B\pi r^2$$

$$\frac{d\Phi}{dt} = 2\pi B r \frac{dr}{dt}$$

v

9)

[6 pts.] A metal bar of height L is moving and accelerating towards the right through a uniform magnetic field B pointing out of the page as shown in the picture. The voltage difference between the top and bottom ends of the bar, $\Delta V = V_{top} - V_{bottom}$, is :



positive and constant

negative and constant

zero

positive and increasing

negative and decreasing

oscillating between positive and negative

None of the above

10)

[6 pts.] The current inside a coil is ramped up steadily from 0 to 5 A during a 0.5 s time interval. It is found to induce an EMF of 2 V. What is the inductance of the coil?

20 H

5 H

$$\mathcal{E} = L \frac{\Delta I}{\Delta t}$$

2 H

$$L = 2V / (5A / 0.5s)$$

0.5 H

0.2 H

0.05 H

None of the above

11)

11) [8 pts.] What current is needed in a long solenoid coil (2000 windings, 1.5 m long) to produce an energy density of 20J/m³ inside it?

32 A

17.6 A

4.2 A

3.4 A

3.0 A

0.88 A

None of the above

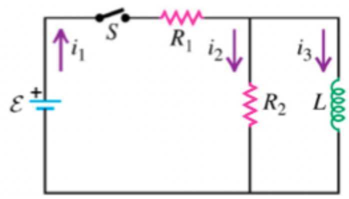
$B = \frac{\mu_0 I N}{L}$, $u = \frac{B^2}{2\mu_0}$

$B = \sqrt{2\mu_0 \cdot u}$
$= 7\,mT$

$I = \frac{BL}{\mu_0 N}$
$\approx 4.2\,A$

12)

12) [10 pts.] In the circuit shown the switch is closed at time t=0. Find the current i_1 immediately after the switch has been closed, $i_1(t \to 0)$, and in the long-time limit, $i_1(t \to \infty)$.



$\frac{\epsilon}{R_1 + R_2}$ and 0

0 and 0

0 and $\frac{\epsilon}{R_1}$

0 and $\frac{\epsilon}{R_2}$

$\frac{\epsilon}{R_1 + R_2}$ and $\frac{\epsilon}{R_1}$

$\frac{\epsilon}{R_1}$ and $\frac{\epsilon}{R_1 + R_2}$ and

None of the above