Solutions

Physics 207 - Exam I

Spring 2020 (520-524; 525-529; 531-535) February 10, 2020.

Please fill out the information and read the instructions below, but
do not open the exam until told to do so.

Rules of the exam:

1. You have 75 minutes (1.25 hrs.) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may not use any other formula sheet.
3. Check to see that there are 6 numbered (3 double-sided) pages plus a blank page for additional work if needed, in addition to the scantron-like cover page. Do not remove any pages.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. Be sure to indicate at the problem under consideration that the extra space is being utilized so the graders know to look at it!
5. **You will be allowed to use only non-programmable calculators on this exam.**
6. **NOTE** that you **must** show your work clearly to receive full credit.
7. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be quite distracting.
8. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given only if your work is legible, clearly explained, and labelled.
9. All of the questions require you show your work and reasoning.
10. Have your TAMU ID ready when submitting your exam to the proctor.

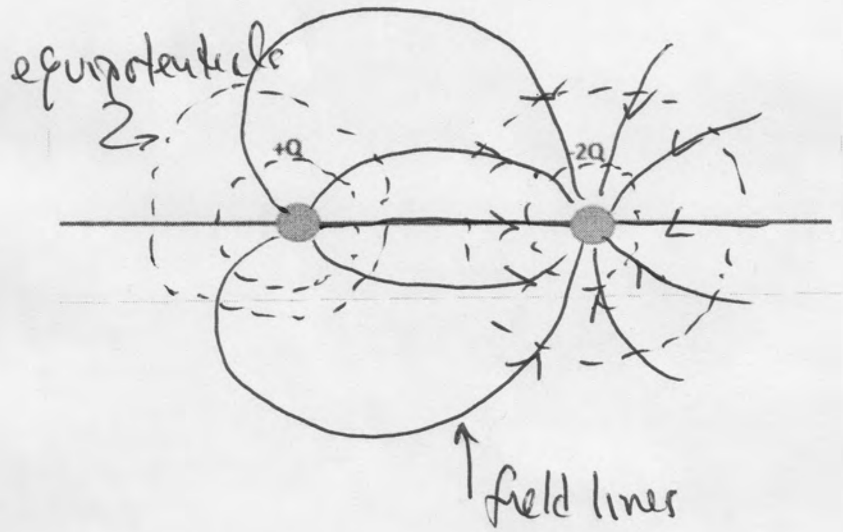
Fill out the information below and sign to indicate your understanding of the above rules

Name:________________________________ UIN:_______________
(Printed legibly)

Signature:________ Section Number: ________

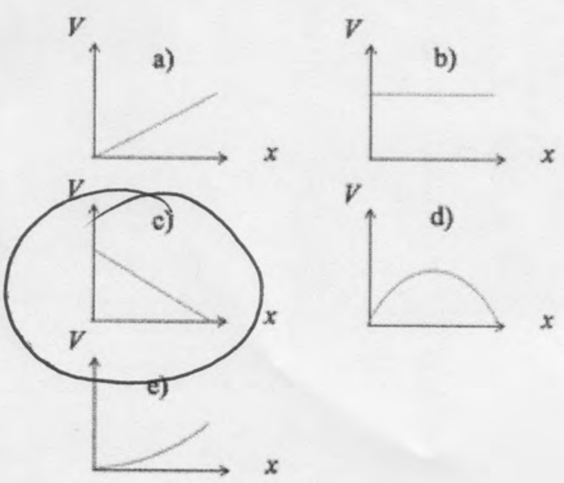
Instructor: McIntyre Ross Webb
(circle one)

A. Consider the charge distribution in the figure below. You are given two spheres of equal diameter one with charge +Q and the other with charge -2Q. For this charge distribution, sketch the electric field lines in the vicinity of the charges using solid lines for the electric field lines. In addition, sketch at least 3 equipotential surfaces in the vicinity of each of the charges using dashed lines.



LO	P	F
13.1		
27.1		

B. A region of space contains a uniform electric field, with a constant magnitude E and directed along the positive x-axis. Which figure below could correctly describe the electric potential as a function of x? You must explain your choice in words or with an equation to receive credit.



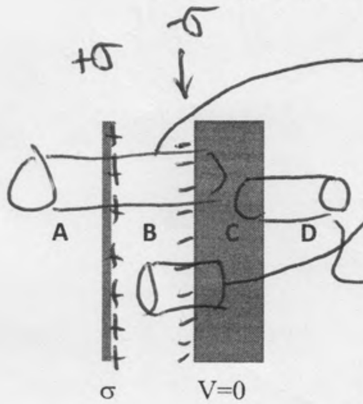
Because

$$\Delta V = -\int \vec{E} \cdot d\vec{\ell} = -Ex$$

$$V(x) = -Ex + V(ref)$$

LO	P	F
5.1		
22.1		
26.1		

C. You are given a collection of two parallel infinite planes. The left plane is a thin insulator carrying a charge per unit area, σ, and the rightmost plane is a thick conducting plane connected to "ground" (V=0).
Find the electric field in each of the labeled regions below. In terms of the quantities given, find the surface charge density on the conducting plane and sketch on the figure below where this charge resides.



A – $E_{TOTAL} = \frac{\sigma A - \sigma A}{\epsilon_0} = 0$

B – $E = \frac{\sigma}{\epsilon_0}$ to the right

C – $E = 0$ inside a conductor

D – $E = \frac{0}{\epsilon_0} = 0$

$-\sigma$ appears on left surface of conducting sheet.

LO	P	F
2.1		
2.2		
2.3		
3.1		
16.1		
18.1		
18.2		
18.3		
19.1		
19.2		
~~25.1~~		

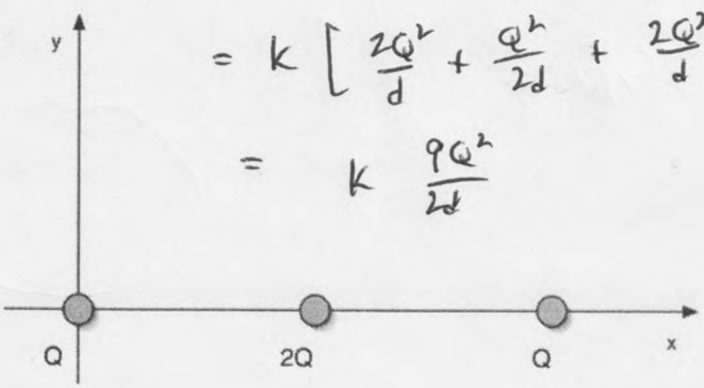
D. A system of three point charges, Q, $2Q$ and Q, are arranged on the x-axis as shown. The spacing between the charges is d. In terms of the quantities given, what is the total energy required to assemble these charges if they are originally very far apart?

$$U = k\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right]$$

$$= k\left[\frac{2Q^2}{d} + \frac{Q^2}{2d} + \frac{2Q^2}{d}\right]$$

$$= k\,\frac{9Q^2}{2d}$$

LO	P	F
3.2		
20.1		
20.2		
20.3		



Problem I.

You are given two point charges of equal but opposite charge, Q. The positive charge is located at (0,3d) and the negative charge is located at (-4d,0). In terms of the quantities given, answer the following:

A. Find the force on the positive charge due to the negative charge. (magnitude and direction or components)

B. Find the E-field due to these charges at the origin. (magnitude and direction or components)

C. How much work was done by an external force in bringing the positive charge in from infinity after the negative charge was in its final position?

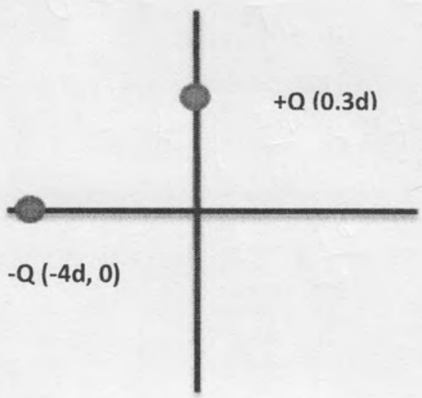
D. Using V=0 at r = ∞ as our reference potential, find the potential due to these two charges at the origin.

A) $\vec{F}_{Coulomb} = \frac{k Q_1 q_2}{r_{12}^2}\hat{r}_{12} = -\frac{kQ}{(5d)^2}\left[\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right]$

B) $\vec{E}_{TOTAL}(origin) = \vec{E}_1(0,0) + \vec{E}_2(0,0) = -kQ\left[\frac{1}{(4d)^2}\hat{i} + \frac{1}{(3d)^2}\hat{j}\right]$

C) $Work = -\Delta U = -\left[-\frac{kQ^2}{5d}\right]$

D) $V(0,0) = V_1(0,0) + V_2(0,0) = \frac{kQ}{3d} - \frac{kQ}{4d}$



LO	P	F
2.4		
2.5		
3.3		
8.1		
10.1		
11.1		
20.4		
21.1		
23.1		

Problem II.

A *very long* solid insulating cylinder with radius R has a positive charge uniformly distributed throughout its volume, with a positive charge per unit length $+\lambda$. In addition, a very long thin insulating wire is located along the axis of the charged cylinder, with the wire carrying a uniform negative charge per unit length, $-\lambda$. In terms of the quantities given, answer the following for a position far from the ends of the wire and cylinder:

A. Find the volume charge density, ρ, in the solid cylinder that results in a charge per unit length, λ , for the insulating cylinder.

B. Starting from a formula on the formula sheet, derive an expression for the electric field inside the volume of the cylinder at a distance of $r < R$ from the axis of the cylinder due to both the wire and the cylinder. (You must show work to get full credit.)

C. Derive a similar expression for the electric field outside the volume of the cylinder for $r > R$. (You must show work to get full credit.)

A) 

$$q_{total} = \lambda L = \rho\,[\text{volume}] = \rho(\pi R^2 L)$$

solving for $\rho = \lambda / \pi R^2$

B) using Gauss's law $\oint \vec{E}\cdot d\vec{A} = q_{enc}/\epsilon_0$

$$E(r)(2\pi r L) = \frac{[-\lambda L + \rho \pi r^2 L]}{\epsilon_0}$$

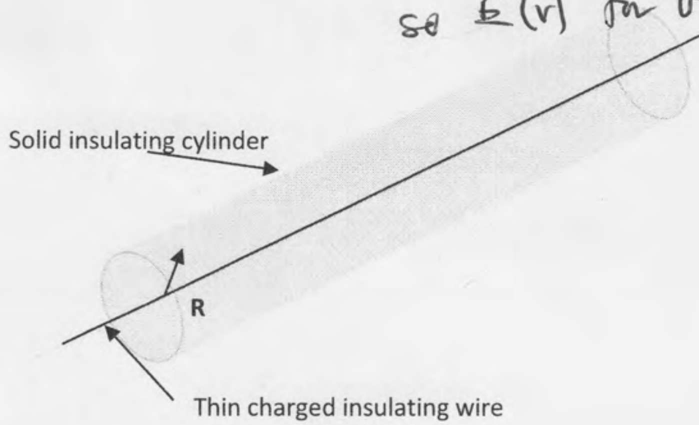
$$E(r) = \frac{-\lambda L + \rho\pi r^2 L}{2\pi\epsilon_0 r L} = \frac{-\lambda + \rho\pi r^2}{2\pi\epsilon_0 r}$$ radially inward

C) Again using Gauss's law

$$\oint \vec{E}\cdot dA = q_{enc}/\epsilon_0 = \frac{-\lambda L + \lambda L}{\epsilon_0} = 0$$

so $E(r)$ for $r > R$ $= 0$

LO	P	F
3.4		
5.2		
~~7.1~~		
~~7.2~~		
15.1		
16.2		
18.4		
18.5		



Problem III.

You are given a uniformly charged thin insulating rod of length, *L*, with a total charge, *Q*. This charged rod is located on the y-axis as shown in the figure. Taking the potential at infinity to be zero, answer the following in terms of the quantities given.

A. What is the charge per unit length of this insulating rod?

B. Starting from the definition of the potential for a continuous charge distribution, derive an expression for the electric potential for point **a** on the y-axis for y>0. You must show work to get full credit.

A) $\lambda = Q/L$

B) $V(r) = \int \frac{k\,dq'}{|\vec{r}-\vec{r}'|}$ with $dq' = \lambda\,dy'$; $\vec{r} = y\hat{j}$; $\vec{r}' = y'\hat{j}$

LO	P	F
3.5		
5.3		
5.4		
7.3		
22.2		

$$V(y) = \int_{-0}^{-L} \frac{k\lambda\,dy'}{(y-y')} = k\lambda \ln(y-y')\Big|_{-0}^{-L}$$

$$= k\lambda\left[\ln(y+L) - \ln(y)\right]$$

$$= k\lambda \ln\left(y+L/y\right)$$